Durbin-Watson test

A test that the residuals from a linear regression or multiple regression are independent.

Method:

Because most regression problems involving time series data exhibit positive autocorrelation, the hypotheses usually considered in the Durbin-Watson test are

$$H_0 : \rho = 0$$
$$H_1 : \rho > 0$$

The test statistic is

$$d = \frac{\sum_{i=2}^{n}(e_i - e_{i-1})^2}{\sum_{i=1}^{n} e_i^2}$$

where $e_i = y_i - \hat{y}_i$ and y_i and $\hat{y}_i$ are, respectively, the observed and predicted values of the response variable for individual i. d becomes smaller as the serial correlations increase. Upper and lower critical values, d_U and d_L have been tabulated for different values of k (the number of explanatory variables) and n.

If $d < d_L$	reject $H_0 : \rho = 0$
If $d > d_U$	do not reject $H_0 : \rho = 0$
If $d_L < d < d_U$	test is inconclusive.

Example:

TABLE1. Data for Soft Drink Concentrate Sales Example

	t	(1) Annual Regional Concentrate Sales(units) y_t	(2) Annual Advertising Expenditures (\$×1000) x_t	(3) Least-Square Residuals e_t	(4) e_t^2	(5) $(e_t - e_{t-1})^2$	(6) Annual Regional Population z_t
1960	1	3083	75	−32.330	1045.2289		825000
1961	2	3149	78	−26.603	707.7196	32.7985	830445
1962	3	3218	80	2.215	4.9062	830.4471	838750
1963	4	3239	82	−16.967	287.8791	367.9491	842940
1964	5	3295	84	−1.148	1.3179	250.2408	846315
1965	6	3374	88	−2.512	6.3101	1.8605	852240
1966	7	3475	93	−1.967	3.8691	0.2970	860760
1967	8	3569	97	11.669	136.1656	185.9405	865925
1968	9	3597	99	−0.513	0.2632	148.4011	871640
1969	10	3725	104	27.032	730.7290	758.7270	877745
1970	11	3794	109	−4.422	19.5541	989.3541	886520
1971	12	3959	115	40.032	1602.5610	1976.1581	894500
1972	13	4043	120	23.577	555.8749	270.7670	900400
1973	14	4194	127	33.940	1151.9236	107.3918	904005
1974	15	4318	135	−2.787	7.7674	1348.8725	908525
1975	16	4493	144	−8.606	74.0632	33.8608	912160
1976	17	4683	153	0.575	0.3306	84.2908	917630
1977	18	4850	161	6.848	46.8951	39.3505	922220
1978	19	5005	170	−18.971	359.8988	666.6208	925910
1979	20	5236	182	−29.063	844.6580	101.8485	929610
		$\sum_{t=1}^{20} e_t^2 = 7587.9154$		$\sum_{t=2}^{20}(e_t - e_{t-1})^2 = 8195.2065$			

We will also use the Durbin-Watson test for

$$H_0 : \rho = 0$$

$$H_1 : \rho > 0$$

$$d = \frac{\sum_{t=2}^{20}(e_t - e_{t-1})^2}{\sum_{t=1}^{20} e_t^2} = \frac{8195.2065}{7587.9154} = 1.08$$

If we choose $\alpha = 0.05$, then Table 2 gives the critical values corresponding to $n = 20$ and one regressor as $d_L = 1.20$ and $d_U = 1.41$.

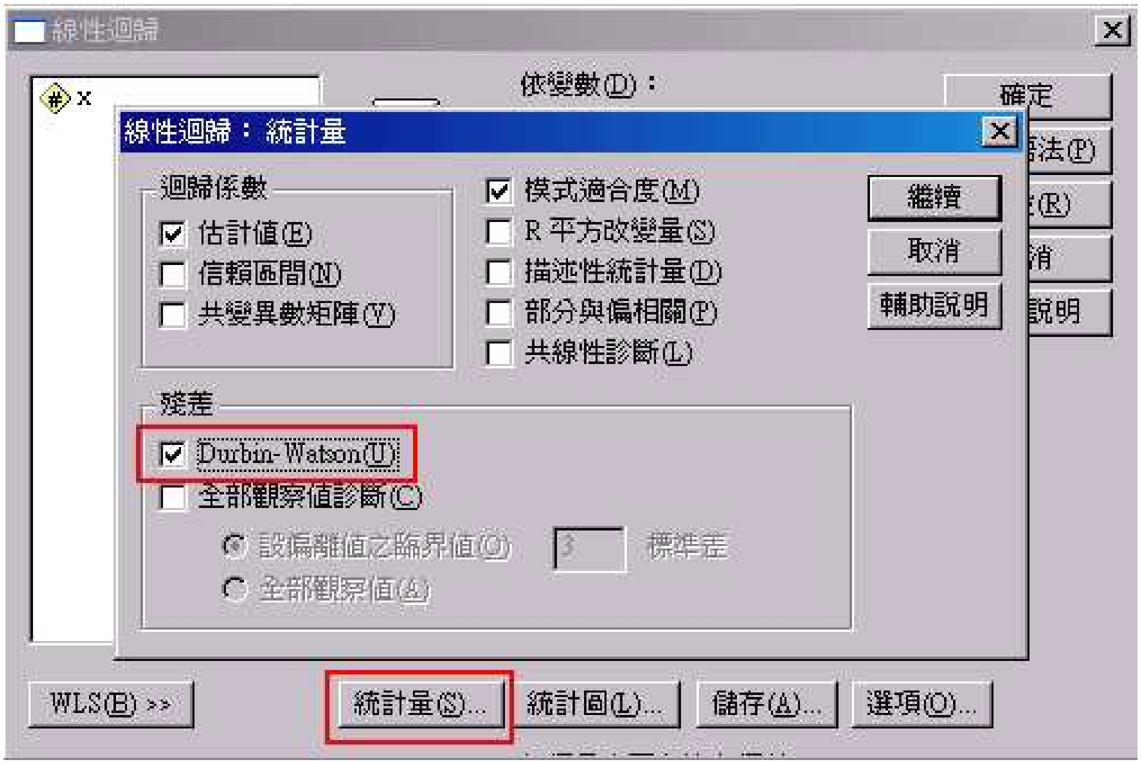
$\because d = 1.08 < d_L = 1.20$

$\therefore$ We reject H_0 and conclude that the errors are positively autocorrelated.

TABLE2. Critical Values of the Durbin-Watson Statistic

Sample Size	Probability in Lower Tail (Significance Level= α)	k = Number of Regressors (Excluding the Intercept) 1		2		3		4		5	
		d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
15	.01	.81	1.07	.70	1.25	.59	1.46	.49	1.70	.39	1.96
	.025	.95	1.23	.83	1.40	.71	1.61	.59	1.84	.48	2.09
	.05	1.08	1.36	.95	1.54	.82	1.75	.69	1.97	.56	2.21
20	.01	.95	1.15	.86	1.27	.77	1.41	.63	1.57	.60	1.74
	.025	1.08	1.28	.99	1.41	.89	1.55	.79	1.70	.70	1.87
	.05	1.20	1.41	1.10	1.54	1.00	1.68	.90	1.83	.79	1.99
25	.01	1.05	1.21	.98	1.30	.90	1.41	.83	1.52	.75	1.65
	.025	1.13	1.34	1.10	1.43	1.02	1.54	.94	1.65	.86	1.77
	.05	1.29	1.45	1.21	1.55	1.12	1.66	1.04	1.77	.95	1.89
30	.01	1.13	1.26	1.07	1.34	1.01	1.42	.94	1.51	.88	1.61
	.025	1.25	1.38	1.18	1.46	1.12	1.54	1.05	1.63	.98	1.73
	.05	1.35	1.49	1.28	1.57	1.21	1.65	1.14	1.74	1.07	1.83
40	.01	1.25	1.34	1.20	1.40	1.15	1.46	1.10	1.52	1.05	1.58
	.025	1.35	1.45	1.30	1.51	1.25	1.57	1.20	1.63	1.15	1.69
	.05	1.44	1.54	1.39	1.60	1.34	1.66	1.29	1.72	1.23	1.79
50	.01	1.32	1.40	1.28	1.45	1.24	1.49	1.20	1.54	1.16	1.59
	.025	1.42	1.50	1.38	1.54	1.34	1.59	1.30	1.64	1.26	1.69
	.05	1.50	1.59	1.46	1.63	1.42	1.67	1.38	1.72	1.34	1.77
60	.01	1.38	1.45	1.35	1.48	1.32	1.52	1.28	1.56	1.25	1.60
	.025	1.47	1.54	1.44	1.57	1.40	1.61	1.37	1.65	1.33	1.69
	.05	1.55	1.62	1.51	1.65	1.48	1.69	1.44	1.73	1.41	1.77
80	.01	1.47	1.52	1.44	1.54	1.42	1.57	1.39	1.60	1.36	1.62
	.025	1.54	1.59	1.52	1.62	1.49	1.65	1.47	1.67	1.44	1.70
	.05	1.61	1.66	1.59	1.69	1.56	1.72	1.53	1.74	1.51	1.77
100	.01	1.52	1.56	1.50	1.58	1.48	1.60	1.45	1.63	1.44	1.65
	.025	1.59	1.63	1.57	1.65	1.55	1.67	1.53	1.70	1.51	1.72
	.05	1.65	1.69	1.63	1.72	1.61	1.74	1.59	1.76	1.57	1.78

Operation of SPSS:



模式摘要[b]

模式	R	R 平方	調過後的 R 平方	估計的標準誤	Durbin-Watson 檢定
1	1.000[a]	.999	.999	20.53165	1.080

a. 預測變數：(常數), X

b. 依變數\：Y

Details:

- The fundamental assumptions in linear regression are that the error terms ε_i have mean zero and constant variance and uncorrelated [$E(\varepsilon_i) = 0$, $\text{Var}(\varepsilon_i) = \sigma^2$, and $E(\varepsilon_i \varepsilon_j) = 0$]. For purposes of testing hypotheses and constructing confidence intervals we often add the assumption of normality, so that the ε_i are NID(0, σ^2). Some applications of regression involve regressor and response variables that have a natural sequential order over time. Such data are called **time series data**. Regression models using time series data occur relatively often in economics, business, and some fields of engineering. The assumption of uncorrelated or independent errors for time series data is often not appropriate. Usually the errors in time series data exhibit **serial correlation**, that is, $E(\varepsilon_i \varepsilon_j \neq 0)$. Such error terms are said to be **autocorrelated**.

- Durbin-Waston test is based on the assumption that the errors in the regression model are generated by a **first-order autoregressive process** observed at equally spaced time periods, that is,

$$\varepsilon_t = \rho \varepsilon_{t-1} + a_t$$

  where ε_t is the error term in the model at time period t, a_t is an NID(0, σ_a^2) random variable, and $\rho(|\rho| < 1)$ is the **autocorrelation parameter**. Thus, a simple linear regression model with **first-order autoregressive errors** would be

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$$
$$\varepsilon_t = \rho\varepsilon_{t-1} + a_t$$

where y_t and x_t are the observations on the response and regressor variables at time period t.

- Situations where negative autocorrelation occurs are not often encountered. However, if a test for negative autocorrelation is desired, one can use the statistic $4-d$. Then the decision rules for $H_0 : \rho = 0$ versus $H_1 : \rho < 0$ are the same as those used in testing for positive autocorrelation. It is also possible to conduct a two-side test ($H_0 : \rho = 0$ versus $H_1 : \rho \neq 0$) by using both one-side tests simultaneously. If this is done, the two-side procedure has Type I error 2α, where α is the Type I error used for each one-side test.

Reference:

1. Montgomery, D. C., Peck, E. A. and Vining, G. G. (2001). Introduction to Linear Regression Analysis. 3rd Edition, New York, New York: John Wiley & Sons.