GENERALIZED CONE-CONTINUITY OF SET-VALUED MAPS WITH SCALARIZATION

PREMYUDA DECHBOON, TAMAKI TANAKA*
BURAPHA UNIVERSITY IN THAILAND, NIIGATA UNIVERSITY IN JAPAN

A composite function is a function which is the nesting of two or more functions to form a single new function. Such operation frequently preserves several mathematical properties of each nested function. For instance, a composition of continuous maps is continuous on topological spaces. From the view point of vector optimization and set optimization, this kind of inheritance by composite operations is important and useful to prove extended results and to get characterizations of optimal solutions through scalarization. This is a typical approach by which optimization problems with vector-valued or set-valued maps can be easily handled by converting vectors or sets into real numbers; see [1] and [3, 2].

Recently, Ike, Liu, Ogata and Tanaka [4] show certain results on the inheritance property of some kinds of continuity of set-valued maps via scalarization functions for sets: if a set-valued map has a kind of continuity (lower continuity or upper continuity; see [2]) then the composition of its set-valued map and a certain scalarization function assures a similar semicontinuity to its scalarization function defined on the family of nonempty subsets of a real topological vector space. Their results are generalizations of results in earlier study by Kuwano, Tanaka and Yamada [6]. However, the statements of inheritance in [4] are confined to four types out of the six set-relations proposed by Kuroiwa, Tanaka and Ha [5]. On the other hand, Sonda, Kuwano, and Tanaka [7] introduce two kinds of continuity with respect to cone, called "cone continuity," for set-valued maps by analogy with semicontinuity for real-valued functions, and they investigate the inheritance properties on cone continuity of parent set-valued maps via scalarization. Therefore, it is interesting to investigate the inheritance of cone continuity for set-valued maps via general scalarization functions for sets in the same manner as [4].

The aim of this talk is to intrduce the mechanism by which composite functions of a set-valued map and a scalarization function transmit semicontinuity of parent set-valued maps through several scalarization for sets.

References

[1] C. Gerth (Tammer) and P. Weidner, *Nonconvex separation theorems and some applications in vector optimization*, J. Optim. Theory Appl., **67** (1990), 297–320.

[2] A. Göpfert, H. Riahi, C. Tammer, and C. Zălinescu, *Variational methods in partially ordered spaces*, vol. 17 of CMS Books in Mathematics, Springer-Verlag, New York, 2003.

[3] A. Hamel and A. Löhne, *Minimal element theorems and Ekeland's principle with set relations*, J. Nonlinear Convex Anal., **7** (2006), 19–37.

[4] K. Ike, M. Liu, Y. Ogata, and T. Tanaka, *Semicontinuity of the composition of set-valued map and scalarization function for sets*, J. Appl. Numer. Optim., **1** (2019), 267–276.

[5] D. Kuroiwa, T. Tanaka, and T. X. D. Ha, *On cone convexity of set-valued maps*, in Proceedings of the Second World Congress of Nonlinear Analysts, Part 3 (Athens, 1996), Nonlinear Anal., **30** (1997), 1487–1496.

[6] I. Kuwano, T. Tanaka, and S. Yamada, *Unified scalarization for sets and set-valued Ky Fan minimax inequality*, J. Nonlinear Convex Anal., **11** (2010), 513–525.

[7] Y. Sonda, I. Kuwano, and T. Tanaka, *Cone-semicontinuity of set-valued maps by analogy with real-valued semicontinuity*, Nihonkai Math. J., **21** (2010), 91–103.

(P.Dechboon) Burapha University, Chonburi 20131, Thailand

(T.Tanaka) Niigata University, Niigata 950–2181, Japan.
Email address: tamaki@math.sc.niigata-u.ac.jp

*Presenting author.