國立中山大學應用數學系

學術演講

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講　題：Generalized list colouring of graphs (Ⅰ), (Ⅱ)

時　間：2020/2/3（Monday）14:10 ~ 16:00

地　點：理學院四樓理 SC 4009-0 多功能互動教室

茶　會：13:30 於理 SC 4010 室 (系辦公室)

Abstract

Assume $\mathcal{G}$ is a hereditary family of graphs, i.e., if $G \in \mathcal{G}$ and H is a subgraph of G, then $H \in \mathcal{G}$. An $\mathcal{G}$-colouring of a graph G is a colouring ϕ of the vertices of G so that each colour class induces a graph in $\mathcal{G}$. An $\mathcal{G}$-n-colouring of G is an $\mathcal{G}$-colouring ϕ of G such that $\phi(v) \in [n] = \{1, 2, \ldots, n\}$ for each vertex v. We say G is $\mathcal{G}$-n-colourable if there exists an $\mathcal{G}$-n-colouring of G. The $\mathcal{G}$-chromatic number of G is

$$\chi_{\mathcal{G}}(G) = \min\{n : G \text{ is } \mathcal{G}\text{-}n\text{-colourable}\}.$$

Assume L is a list assignment of G. An $\mathcal{G}$-L-colouring of G is an $\mathcal{G}$-colouring ϕ of G so that $\phi(v) \in L(v)$ for each vertex v. We say G is $\mathcal{G}$-n-choosable if for every n-list assignment L of G, there exists a $\mathcal{G}$-L-colouring of G, The $\mathcal{G}$-*choice number* of G is

$$ch_{\mathcal{G}}(G) = \min\{n : G \text{ is } \mathcal{G}\text{-}n\text{-choosable}\}.$$

Many colouring concepts studied in the literature are $\mathcal{G}$-colourings for special graph families $\mathcal{G}$.

We denote by

- $\mathcal{G}_k$ the family of graphs whose connected components are of order at most k;
- $\mathcal{D}_k$ the family of graphs of maximum degree at most k;
- $\mathcal{F}$ the family of forests;

- $\mathcal{S}$ the family of star forests;
- $\mathcal{L}$ the family of linear forests;
- $\mathcal{C}_k$ the family of graphs of colouring number at most k.
- $\mathcal{M}_k$ the family of graphs of maximum average degree at most k.

Many of the $\mathcal{G}$-colourings have special names and are studied extensively in the literature.

- An $\mathcal{G}_k$-colouring of G is a colouring of G with clustering k. In particular, an $\mathcal{G}_1$-colouring of G is a proper colouring of G.
- A $\mathcal{D}_k$-colouring of G is a k-defective colouring of G. The parameter $\chi_{\mathcal{D}_k}(G)$ is the *k-defective chromatic number* of G. Also, a $\mathcal{D}_0$-colouring of G is a proper colouring of G.
- An $\mathcal{F}$-colouring of G is a vertex arboreal colouring of G. The parameter $\chi_{\mathcal{F}}(G)$ is the *vertex arboricity* of G, and $ch_{\mathcal{F}}(G)$ is the *list vertex arboricity* of G.
- The parameter $\chi_{\mathcal{S}}(G)$ is the *star vertex arboricity* of G, and $ch_{\mathcal{S}}(G)$ is the *star list vertex arboricity* of G.
- The parameter $\chi_{\mathcal{L}}(G)$ is the *linear vertex arboricity* of G, and $ch_{\mathcal{L}}(G)$ is the *linear list vertex arboricity* of G.

For any two graph families $\mathcal{G}$ and $\mathcal{G}'$, for any graph G, it follows easily from the definition that

$$\chi_{\mathcal{G}}(G) \quad \leq \quad (\max_{H \in \mathcal{G}'} \chi_{\mathcal{G}}(H))\chi_{\mathcal{G}'}(G), \tag{1}$$

and this upper bound is tight. For example,

$$\chi(G) \leq 2\chi_{\mathcal{F}}(G),\ \chi_{\mathcal{S}}(G) \leq 2\chi_{\mathcal{F}}(G) \text{ and } \chi(G) \leq (k+1)\chi_{\mathcal{M}_k}(G),$$

and for any integers k, k',

$$\chi_{\mathcal{G}_k}(G) \leq \left\lceil \frac{k'}{k} \right\rceil \chi_{\mathcal{G}_{k'}}(G),$$

and the inequalities are tight for some graphs G.

It is natural to ask if the same or similar inequalities hold for the corresponding choice number. Some of such inequalities are posed as conjectures or questions in the literature. For example, the following conjecture was proposed in [Wang, B. Wu, Z. Yan and N. Xie, *A*

Weaker Version of a Conjecture on List Vertex Arboricity of Graphs, Graphs and Combinatorics (2015) 31:1779–1787]:

Conjecture 0.1 *For any graph G,*

$$ch(G) \quad \leq \quad 2ch_{\mathcal{F}}(G).$$

The following question was asked in [Z. Dvořák, J. Pekárek and J. Sereni, *On generalized choice and coloring numbers*, arXiv: 1081.0682403, 2019]:

Question 0.2 *Is it true that for any graph G, for any positive integer k,*

$$ch(G) \leq (k+1)ch_{\mathcal{M}_k}(G)?$$

In this lecture, we disprove Conjecture 0.1 and give a negative answer to Question 0.2. We shall discuss this concept and many open problems concerning this concept.

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