Solutions to Homework Assignment #3

Problem 1. (a) The sample mean is 0.0709 with standard error 0.0322 and t-ratio 2.203. Thus, it is statistically significant at the 5% level. (b) Using dummy variable for Monday, the regression is \( r_t = 0.035 + 0.191 \text{mon}_t + e_t \). The residual ACF gives \( Q(12) = 12.6 \) with p-value 0.32 (\( \chi^2_{11} \)). The residuals appear to be serially uncorrelated, implying that the above regression is adequate. The t-ratio for the Monday coefficient is 2.33. Thus, the Monday effect is significant at the 5% level. [Note that the constant term becomes insignificant. Thus, the regression indicates that the gain in HP stock returns might be explained by Monday effect.]

Problem 2. The simple linear regression with Monday dummy gives \( r_t = 0.081 - 0.100 \text{Mon}_t + e_t \). However, the residuals ACF shows a significant serial correlation at lag 1. The ACF or EACF of the residuals suggests an MA(1) model for the regression. The refined model becomes \( r_t = 0.091 - 0.1 \text{Mon}_t + a_t + 0.115 a_{t-1} \) with \( \sigma_a = 0.891 \). Residual ACF of the refined model gives \( Q(12) = 15.5 \) with p-value 0.115 (\( \chi^2_{10} \)). The refined model is adequate. The t-ratio of the Monday dummy coefficient is \(-8.16\), indicating that the Monday effect is significantly negative at the 5% level.

Problem 3. Because the correlation between \( y_t \) and \( x_t \) is 0.389, I start with the simple linear regression model

\[
y_t = \beta x_t + e_t.
\]

The fitted model is \( y_t = 0.621 x_t + e_t \). The residuals, however, show strong serial dependence, e.g. \( Q(10) = 115.0 \). I then use EACF to identify an ARMA model for the residual and it suggests clearly an ARMA(1,1) model. Consequently, the model becomes

\[
y_t = 0.723 x_t + \frac{1 - 0.936 B}{1 - 0.820 B} a_t
\]

where all estimates are highly significant and \( \sigma_a = 0.029 \). The residual series has no significant serial correlations because \( Q(10) = 5.5 \), which is low.

Note that if one employs autoregressive models only, then he needs a high order model; see Chapter 8 where an AR(8) model is used. This is a good example for demonstrating the usefulness of using ARMA models. Also, the residuals appear to be heteroscedastic. One can further refine the model by using the models discussed in Chapter 3.