1. (10分) Show that the function

\[ f(x) = \begin{cases} 
  x \sin \frac{1}{x}, & x \neq 0, \\
  0, & x = 0,
\end{cases} \]

is continuous on \( \mathbb{R} \) but not differentiable at \( x = 0 \).

2. (10分) Let \( n \times n \) matrix \( A \) have all entries 1. Find all of its eigenvalues and corresponding eigenvectors.

3. (15分) Let \( \{z_n\} \) be a sequence in \( \mathbb{R} \). If \( z_n \to \alpha \), show that

\[ \frac{z_1 + z_2 + \cdots + z_n}{n} \to \alpha. \]

4. (15分) Assume \( f : \mathbb{R}^n \to \mathbb{R} \) is a function whose partial derivatives of order \( \leq 2 \) are everywhere defined and continuous. Let \( a \in \mathbb{R}^n \) be a critical point of \( f \) (i.e. \( \frac{\partial f}{\partial x_i}(a) = 0, i = 0, 1, 2, \ldots, n \)). Prove that \( a \) is a local minimum provided the Hessian matrix

\[ \left( \frac{\partial^2 f}{\partial x_i \partial x_j} \right) \]

is positive definite at \( x = a \).

5. (15分) Prove the Second Fundamental Theorem of Calculus:

Let \( f \) be continuous on an interval \( I \) and define \( F(x) = \int_a^x f(t)dt \) for some \( a \in I \). Then \( F'(x) = f(x) \) for each \( x \) in \( I \).
6. (15分) Let \( T : V \rightarrow W \) be a linear transformation between finite-dimensional vector spaces. Prove that
\[
\dim(\ker T) + \dim(\text{range } T) = \dim V.
\]

7. Let \( A = (a_{ij}), B = (b_{ij}) \) be \( n \times n \) matrices where
\[
a_{ij} = \begin{cases} 
0 & \text{if } i = j, \\
1 & \text{if } i \neq j,
\end{cases}
\]
\[
b_{ij} = \begin{cases} 
2 & \text{if } i = j = 1, \\
0 & \text{otherwise}.
\end{cases}
\]

(a) (10分) Find the characteristic polynomial of \( A \).

(b) (10分) Find \( \det(A + B) \).

End.