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Star-extremal graphs and the lexicographic product

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Abstract

The star-chromatic number and the fractional-chromatic number are two generalizations of the ordinary chromatic number of a graph. We say a graph G is *star-extremal* if its star-chromatic number is equal to its fractional-chromatic number. We prove that star-extremal graphs G have the following interesting property: For an arbitrary graph H the star-chromatic number $\chi^\star(G[H])$ of the lexicographic product $G[H]$ is equal to the product of $\chi^\star(G)$ and $\chi(H)$. Then we show that several classes of circulant graphs are star-extremal. Thus for these circulant graphs G and arbitrary graphs H, if $\chi^\star(G)$ and $\chi(H)$ are known then we can easily determine the star-chromatic number (hence the ordinary chromatic number) of the lexicographic product $G[H]$. For these star-extremal circulant graphs, we also derive polynomial-time anti-clique-finding and coloring algorithms.

1. Introduction

Let k and d be positive integers such that $k \geqslant 2d$. A (k,d)-coloring of a graph $G = (V,E)$ is a mapping $c: V \mapsto Z_k = \{0,1,\ldots,k-1\}$ such that, for each edge $uv \in E$, $|c(u) - c(v)|_k \geqslant d$, where $|x|_k = \min\{|x|, k - |x|\}$. This generalizes the usual notion of a k-coloring: an ordinary k-coloring of G is just a $(k,1)$-coloring of G. Thus, the chromatic number $\chi(G)$ of G is the smallest k for which there is a $(k,1)$-coloring. The *star-chromatic number* of $G, \chi^\star(G)$, is defined as

$$\chi^\star(G) = \inf\{k/d : G \text{ has a } (k,d)\text{-coloring}\}.$$

The star-chromatic number of a graph was first introduced by Vince [13]. His approach uses some methods and results from continuous mathematics. A purely

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