Eleventh Workshop on Numerical Ranges and Numerical Radii

第十一屆數值域與數值半徑研討會

National Sun Yat-sen University, Kaohsiung, Taiwan

台灣・高雄・中山大學

July 9-12, 2012

Special guest: Pei Yuan Wu 吳培元(Taiwan)

Speakers

T. Ando (Japan), N. Bebiano (Portugal), Jor-Ting Chan (Hong Kong), Chi-Tung Chang (Taiwan), Wai-Shun Cheung (Hong Kong), M. D. Choi (Canada), Daeshik Choi (USA), Mao-Ting Chien (Taiwan), Chang-Pao Chen (Taiwan), M. Fiedler (Czech), Hwa-Long Gau (Taiwan), Anne Greenbaum (USA), John Holbrook (Canada), Nathaniel Johnston (Canada), David W. Kribs (Canada), Tsang-Hai Kuo（Taiwan）, Hang-Chin Lai (Taiwan), Denny H. Leung (Singapore), Chi-Kwong Li (USA), H. Nakazato (Japan), Chi-Kueng Ng (China), Yiu Tung Poon (USA) , Agnes Radl (Switzerland), A. Salemi (Iran), Ana Nata Santos (Portugal), Mau-Hsiang Shih (Taiwan), Raymond Nung-Sing Sze (USA), Wataru Takahashi (Japan), Bit-Shum Tam (Taiwan), Tin Yau Tam (USA), Ming-Cheng Tsai (Taiwan), Frank Uhlig (USA), Batzorig Undrakh (Mongolia), Hao-Wei Huang (USA), Kuo-Zhong Wang (Taiwan), Ya-Shu Wang (Taiwan), , J. Zemanek (Poland) , F. Zhang (USA), K. Zyczkowski (Poland).

Sponsors and Endorsers: National Sun Yat-sen University (NSYSU), Taiwan National Science Council (NSC), and the International Linear Algebra Society (ILAS)

Organizing Committee: **Mao-Ting Chien 簡茂丁, Hwa-Long Gau 高華隆, Chi-Kwong Li 李志光, Ying-Fen Lin 林英芬, Ngai-Ching Wong 黃毅青, and Pei Yuan Wu 吳培元**

The Eleventh Workshop on Numerical Ranges and Numerical Radii

第十一屆數值域與數值半徑研討會

Science Building 3001, NSYSU, Taiwan.

As of 2012/7/3

	Monday July 9	Tuesday July 10	Wednesday July 11	Thursday July 12
8:50-9:20am	**Registration/Opening**	Miroslav Fiedler	Tsuyoshi Ando	Jaroslav Zemanek
9:25-9:55	Man-Duen Choi	Fuzhen Zhang	Mau-Hsiang Shih	Natália Bebiano
10:00-10:30	Chi-Keung Ng	Hiroshi Nakazato	Wataru Takahashi	Hao-Wei Huang
	Tea/coffee/snacks			
11:00-11:30	Denny H. Leung	Batzorig Undrakh	Yiu Tung Poon	Tin Yau Tam
11:35-12:05	Ya-Shu Wang	Frank Uhlig	Nathaniel Johnston	Daeshik Choi
12:10-12:40	Jor-Ting Chan	John Holbrook	Pei Yuan Wu	Chi-Kwong Li
	Lunch			**Sightseeing/Boat tour (12:00 - right after the last talk)**
1:50-2:20pm	Karol Zyczkowski	Abbas Salemi	Tsang-Hai Kuo	
2:25-2:55	Raymond N-S Sze	Ana Nata Santos	Hwa-Long Gau	
	Tea/coffee/snacks			
3:25-3:55	Agnes Radl	Anne Greenbaum	Kuo-Zhong Wang	
4:00-4:30	Wai-Shun Cheung	David W. Kribs	Ming-Cheng Tsai	
	Tea/coffee/snacks	Ming-Hsiu Hsu (PhD defense, 4:30-6:30)	Tea/coffee/snacks	
4:45-5:15	Chang-Pao Chen		Chi-Tung Chang	
5:20-5:50	Mao-Ting Chien		Hang-Chin Lai	
6:00	**Group Photo**			
6:30	**Reception (in campus)**		**Concert/Banquet (Uni Resort Hotel)**	

1. **Registration/Opening: 8:50-9:20 am at venue; collecting registration (US$100) and tour (US$30).**
2. Group Photo will be taken at the bronze statue of Dr. Sun Yat-sen in the campus garden.
2. Reception will be held in the Chinese restaurant in the Student Union Building.
3. A Chinese traditional music concert performed by local school children is held before the banquet.
4. We will celebrate the 65th birthday of Professor Pei-Yuan Wu during the banquet. Prepare to speak or sing.

THE ELEVENTH WORKSHOP ON NUMERICAL RANGES AND NUMERICAL RADII
(WONRA2012)
第十一屆數值域與數值半徑研討會

SCIENCE BUILDING 3001
DEPARTMENT OF APPLIED MATHEMATICS
NATIONAL SUN YAT-SEN UNIVERSITY
KAOHSIUNG, TAIWAN, R. O. C.
JULY 9–12, 2012.

as of July 3, 2012

Monday, July 9, 2012

08:50 – 09:20 Registration/Opening
Hong-Kun Xu (National Sun Yat-sen University, Taiwan)
Chi-Kwong Li (College of William and Mary, USA)

(Chair: Ngai-Ching Wong)

09:25 – 09:55 Man-Duen Choi (University of Toronto, Canada) (page 11)
The panorama of two by two complex matrices.

10:00 – 10:30 Chi-Keung Ng (Nankai University, China) (page 17)
A Murray-von Neumann type classification of C^*-algebras.

TEA/COFFEE/SNACKS

(Chair: Hwa-Long Gau)

11:00 – 11:30 Denny H. Leung (National University of Singapore, Singapore) (page 16)
Linear and nonlinear disjointness preserving operators on function spaces.

11:35 – 12:05 Ya-Shu Wang (National Central University, Taiwan) (page 22)
Preservers on the Lipschitz functions.

12:10 – 12:40 Jor-Ting Chan (University of Hong Kong, Hong Kong) (page 8)
Linear preservers of the joint numerical radius.

Lunch

(Chair: Denny Leung)

13:50 – 14:20 Karol Zyczkowski (Jagiellonian University Cracow, Poland) (page 24)
Numerical shadow: a probability measure supported by the numerical range.

14:25 – 14:55 Raymond N-S Sze (The Hong Kong Polytechnic University, Hong Kong) (page 20)
The (p, k) matricial ranges and operator quantum error correction.

TEA/COFFEE/SNACKS

(Chair: Frank Uhlig)

15:25 – 15:55 Agnes Radl (Universitaet Bern, Switzerland) (page 18)
The numerical range of positive operators.

16:00 – 16:30 Wai-Shun Cheung (University of Hong Kong, Hong Kong) (page 10)
Elementary Proofs For Some Numerical Range Results.

TEA/COFFEE/SNACKS

(Chair: Tin Yau Tam)

16:45 – 17:15 Chang-Pao Chen (Hsuan Chuang Univrersity, Taiwan) (page 9)
The Muckenhoupt-type estimations for the best constants in multidimensional modular inequalities over spherical cones.

17:20 – 17:50 Mao-Ting Chien (Soochow University, Taiwan) (page 10)
Determinantal representation of trigonometric polynomials.

Group Photo taken at the bronze statue of Dr. Sun Yat-sen in campus garden (18:00 pm)

Reception at the Chinese Restaurant in the Student Union Building (18:30 pm)

Tuesday, July 10, 2012

(`Chair: Chi-Kwong Li`)

08:50 – 09:20 Miroslav Fiedler (Institute of Computer Science ASCR, Czech) (page 11)
Factorizable matrices.

09:25 – 09:55 Fuzhen Zhang (Nova Southeastern University, USA) (page 23)
Some inequalities of majorization type.

10:00 – 10:30 Hiroshi Nakazato (Hirosaki University, Japan) (page 17)
Numerical range associated with a closed orbit under a central force.

TEA/COFFEE/SNACKS

(`Chair: Fuzhen Zhang`)

11:00 – 11:30 Batzorig Undrakh (National University of Mongolia, Mongolia) (page 10)
On the numerical range of the weighted shift operators.

11:35 – 12:05 Frank Uhlig (Auburn University, USA) (page 21)
Fields of values for matrix factorizations and inner $O(n^2)$ approximations of the field of values.

12:10 – 12:40 John Holbrook (University of Guelph, Canada) (page 13)
Compressions of normal matrices.

Lunch

(Chair: Jor-Ting Chan)

13:50 – 14:20 Abbas Salemi (Shahid Bahonar University, Iran) (page 18)
GMRES and polynomial numerical hulls of matrices.

14:25 – 14:55 Ana Nata Santos (Polytechnic Institute of Tomar, Portugal) (page 19)
Remarks on the numerical range of banded biperiodic Toeplitz operators: theory and computer generation.

TEA/COFFEE/SNACKS

(Chair: Miroslav Fiedler)

15:25 – 15:55 Anne Greenbaum (University of Washington, USA) (page 12)
2*-spectral sets and similarity transformations with condition number* 2.

16:00 – 16:30 David W. Kribs (University of Guelph, Canada) (page 15)
Private quantum codes: introduction and connections with quantum error correction.

TEA/COFFEE/SNACKS

(Chair: Man-Duen Choi)

16:30 – 18:30 Ming-Hsiu Hsu (National Sun Yat-sen University, Taiwan) (page 14)
PhD Defense: "Isometries of real and complex Hilbert C-modules".*

Wednesday, July 11, 2012

(`Chair: Jaroslav Zemanek`)

08:50 – 09:20 Tsuyoshi Ando (Hokkaido University, Japan) (page 8)
Quadratic inequalities and factorizations of matrices.

09:25 – 09:55 Mau-Hsiang Shih (National Taiwan Normal University, Taiwan) (page 19)
Construction of Brain computing machines.

10:00 – 10:30 Wataru Takahashi (Tokyo Institute of Technology, Japan) (page 20)
Linear operators in nonlinear analysis and applications.

TEA/COFFEE/SNACKS

(`Chair: Pjek-Hwee Lee`)

11:00 – 11:30 Yiu Tung Poon (Iowa State University, USA) (page 18)
Generalized numerical ranges and quantum error correction.

11:35 – 12:05 Nathaniel Johnston (University of Guelph, Canada) (page 15)
Duality of entanglement norms.

12:10 – 12:40 Pei Yuan Wu (National Chiao Tung University, Taiwan) (page 23)
Diagonals and numerical ranges of finite matrices.

Lunch

Wu's afternoon

(`Chair: Chang-Pao Chen`)

13:50 – 14:20 Tsang-Hai Kuo (Chang Gung University, Taiwan) (page 15)
My acquaintance with Professor Pei-Yuan Wu.

14:25 – 14:55 Hwa-Long Gau (National Central University, Taiwan) (page 12)
Prof. Wu's journey through numerical ranges (Part I).

TEA/COFFEE/SNACKS

(`Chair: Mao-Ting Chien`)

15:25 – 15:55 Kuo-Zhong Wang (National Chiao Tung University, Taiwan) (page 22)
Prof. Wu's journey through numerical ranges (Part II).

16:00 – 16:30 Ming-Cheng Tsai (National Sun Yat-sen University, Taiwan) (page 21)
Prof. Wu's journey through numerical ranges (Part III).

TEA/COFFEE/SNACKS

(`Chair: Yiu Tung Poon`)

16:45 – 17:15 Chi-Tung Chang (National Chiao Tung University, Taiwan) (page 9)
Prof. Wu's journey through numerical ranges (Part IV).

17:20 – 17:50 Hang-Chin Lai (Chung Yuan Christian University, Taiwan) (page 15)
Overview on the Relationship Between Multiplier Operators and Invariant Operators.

Music Concert/Banquet in Uni Resort Hotel (18:30 pm)

Thursday, July 12, 2012

(Chair: Nathaniel Johnston)

08:50 – 09:20 Jaroslav Zemánek (Polish Academy of Sciences, Poland) (page 23)
Numerical range in complex analysis.

09:25 – 09:55 Natália Bebiano (University of Coimbra, Portugal) (page 8)
Numerical ranges of Toeplitz operators with matrix symbols.

10:00 – 10:30 Hao-Wei Huang (Indiana University-Bloomington, USA) (page 13)
Supports and regularity for measures in a free additive convolution semigroup.

TEA/COFFEE/SNACKS

(Chair: Pei Yuan Wu)

11:00 – 11:30 Tin Yau Tam (Auburn University, USA) (page 20)
Connectedness, Hessian and generalized numerical range.

11:35 – 12:05 Daeshik Choi (University of Washington, USA) (page 11)
Crouzeix's conjecture and Diagonally Perturbed Jordan blocks.

12:10 – 12:40 Chi-Kwong Li (College of William and Mary, USA) (page 16)
Optimizing quadratic forms of adjacency matrices, and numerical radii of weighted shifts.

Closing Remarks

A half day tour for local attractions

Abstracts

Quadratic inequalities and factorizations of matrices

Tsuyoshi Ando
Hokkaido University (Emeritus), Japan
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Abstract

For $0 \leq A, C \in M_n$ and $B \in M_n$, we consider two inequalities of Schwarz type:

$$\langle Ax|x\rangle \cdot \langle Cy|y\rangle \geq |\langle Bx|y\rangle|^2 \qquad (x, y \in \mathbb{C}^n), \tag{1}$$

and

$$\langle Ax|x\rangle \cdot \langle Cx|x\rangle \geq |\langle Bx|x\rangle|^2 \qquad (x \in \mathbb{C}^n). \tag{2}$$

When $A = C = I$, the inequalities (1) and (2) mean respectively (norm) $\|B\| \leq 1$ and (numerical radius) $w(B) \leq 1$.
It is well-known that the inequality (1) can be equivalently expressed by existence of a contraction $\|W\| \leq 1$ such that $B = A^{\frac{1}{2}} W C^{\frac{1}{2}}$.
In this talk we ask whether a similar expression can be established for the inequality (2).

Numerical ranges of Toeplitz operators with matrix symbols

Natália Bebiano
Department of Mathematics, University of Coimbra, Portugal
Email: bebiano@mat.uc.pt

Abstract

For Toeplitz operators acting on the Hardy space H^2 with definite or indefinite metric, the respective numerical ranges are investigated. Special attention is paid to the indefinite numerical range of banded 2-Toeplitz operators, which is characterized by performing a reduction to the 2-dimensional underlying space. Classes of tridiagonal 2-Toeplitz operators with indefinite hyperbolical range are studied.

Linear Preservers of the Joint Numerical Radius

Jor-Ting Chan
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Abstract

For a complex Hilbert space H, let $B(H)$ denote the vector space of all bounded linear operators on H and $B(H)^n$ the product of n copies of $B(H)$. For $\mathbf{A} = (A_1, \ldots, A_n) \in B(H)^n$, the *joint numerical range* and the *joint numerical radius* of $\mathbf{A}$ are denoted and defined respectively by

$$W(\mathbf{A}) := \{(\langle A_1 x, x\rangle, \ldots, \langle A_n x, x\rangle) : x \in H \text{ and } \|x\| = 1\},$$

and

$$w(\mathbf{A}) := \sup\{\sqrt{|\lambda_1|^2 + \cdots + |\lambda_n|^2} : (\lambda_1, \ldots, \lambda_n) \in W(\mathbf{A})\}.$$

A linear mapping $T : B(H)^n \to B(H)^n$ is called a *preserver* of the joint numerical radius if $w(T(\mathbf{A})) = w(\mathbf{A})$ for every $\mathbf{A} \in B(H)^n$. Recently, Li and Poon obtained a complete description for such T when H is finite-dimensional. In this talk, we shall show that if T is surjective, the same description holds when H is infinite-dimensional. Linear preservers relating to the Davis-Wielandt shell will also be discussed. This is a joint work with Kong Chan.

Prof. Wu's Journey Through Numerical Ranges (Part IV) - Numerical Ranges and Geršgorin Discs

Chi-Tung Chang

Department of Applied Mathematics, National Chiao Tung University, Taiwan

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Abstract

I will introduce the academic contribution of Prof. Pei Yuan Wu about numerical ranges and Geršgorin discs:
For a complex matrix $A = [a_{ij}]_{i,j=1}^n$, let $W(A)$ be its numerical range, and let $G(A)$ be the convex hull of $\bigcup_{i=1}^n \{z \in \mathbb{C} : |z - a_{ii}| \leq (\sum_{i \neq j}(|a_{ij}| + |a_{ji}|))/2\}$ and $G'(A) = \bigcap\{G(U^*AU) : U \text{ } n\text{-by-}n \text{ unitary}\}$. It is known that $W(A)$ is always contained in $G(A)$ and hence in $G'(A)$. In this paper, we consider conditions for $W(A)$ to be equal to $G(A)$ or $G'(A)$. We show that if $W(A) = G'(A)$, then the boundary of $W(A)$ consists only of circular arcs and line segments. If, moreover, A is unitarily irreducible, then $W(A)$ is a circular disc. Complete characterizations of 2-by-2 and 3-by-3 matrices A for which $W(A) = G'(A)$ are obtained. We also give criteria for the equality of $W(A)$ and $G(A)$. In particular, such A's among the permutationally irreducible ones must have even sizes. We also characterize those A's with size 2 or 4 which satisfy $W(A) = G(A)$.

The Muckenhoupt-type estimations for the best constants in multidimensional modular inequalities over spherical cones

Chang-Pao Chen

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Abstract

In this paper, we establish the Muckenhoupt-type estimation for the best constant C associated with the following multidimensional modular inequality over a spherical cone:

$$\left(\int_E \left\{\Phi\left(\int_{\bar{S}_x} k(x,t)f(t)\,d\sigma(t)\right)\right\}^q d\mu\right)^{1/q} \leq C\left(\int_E \{\Phi(f(x))\}^p\,d\nu\right)^{1/p},$$

where $f \in L^p_\Phi(d\nu)$ and $1 \leq p, q \leq \infty$. Similar results are also derived for the complementary integral operator. As a consequence, we give the n-dimensional weighted extensions of Levinson's modular inequality, extensions of Stepanov's and Heinig's results, generalizations of the Hardy-Knopp-type inequalities, and those for the Riemann-Liouville operator and theWeyl fractional operator. We also point out that our estimates are better than the known ones.
This is a joint work of Chang-Pao Chen, Jin-Wen Lan, and Dah-Chin Luor.

Elementary Proofs For Some Numerical Range Results

Wai Shun Cheung

Department of Mathematics, The University of Hong Kong, Hong Kong

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Abstract

From the earliest result of Anderson on the circular symmetry of numerical ranges, to the recent generalization by Wu, the original proofs usually make use of algebraic geometry techniques. We will give elementary proofs which require only simple properties of polynomials and continuous functions.

Determinantal representation of trigonometric polynomials

Mao-Ting Chien

Department of Mathematics, Soochow University, Taiwan

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Abstract

Let A be an $n \times n$ matrix. The numerical range of A is defined as the set $W(A) = \{\xi^* T \xi : \xi \in \mathbf{C}^n, \xi^*\xi = 1\}$. A ternary homogeneous polynomial associated with A defined by $F_A(t,x,y) = \det(tI_n + xH + yK)$ is hyperbolic with respect to $(1,0,0)$, where $H = (A+A^*)/2$, $K = (A-A^*)/(2i)$. It is well known that $W(A)$ is the convex hull of the real affine part of the dual curve of the curve $F_A(t,x,y) = 0$. The Fiedler-Lax conjecture is recently affirmed, namely, for any real ternary hyperbolic form $F(t,x,y)$, there exist real symmetric matrices S_1 and S_2 such that $F(t,x,y) = \det(tI_n + xS_1 + yS_2)$. In this talk, we construct the existence of real symmetric matrices for the ternary hyperbolic forms induced by trigonometric polynomials using Bezoutian and Sylvester elimination methods.

Crouzeix's conjecture and Diagonally Perturbed Jordan blocks

Daeshik Choi
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Abstract

Crouzeix's conjectrue is that for any square matrix A and any polynomial p,

$$\|p(A)\| \leq 2\max\{|p(z)| : z \in W(A)\};$$

where $W(A)$ is the field of values of A and $\|\cdot\|$ denotes the spectral norm. In this paper, we show the conjecture holds for the matrices of the form

$$\begin{pmatrix} \lambda & \alpha_1 & & \\ & \ddots & \ddots & \\ & & \ddots & \alpha_{n-1} \\ \alpha_n & & & \lambda \end{pmatrix},$$

where $\lambda \in \mathbb{C}$ and $\alpha = (\alpha_1, \ldots, \alpha_n) \in \mathbb{C}^n$.

This is a joint work with Anne Greenbaum (U. of Washington).

The panorama of two by two complex matrices

Man-Duen Choi
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Abstract

To attain to the heart of matrix theory, we need to admire the spectacular panorama of 2×2 matrices through the huge picture windows of numerical ranges. More precisely, we look into the down-to-earth information of the numerical ranges of 2×2 matrices, in the structure theory of dilations and positive linear maps, in connection with the recent surprising advances of quantum information. This talk agrees with my personal preference of using 2×2 matrices for theory (but 3×3 matrices for counter-examples).

Factorizable matrices

Miroslav Fiedler
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Abstract

In a series of papers, the author (partially with F. J. Hall) studied matrices obtained by multiplication of simpler matrices, each differing from the identity matrix by one diagonal block, with some restrictions. It turned out the resulting products have intriguing properties. All of them (with fixed factors) have the same spectrum independently of their ordering, they have certain zero - nonzero shapes, certain submatrices of lower rank, etc. The usual companion matrix of a polynomial belongs to such kind of matrices, and this fact led to the discovery of other simple companion matrices.
In the talk, we intend to survey the known results and add a few recent observations.

Prof. Wu's Journey Through Numerical Ranges (Part I)

Hwa-Long Gau
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Abstract

In this talk, I will introduce the academic contribution of Prof. Pei Yuan Wu about numerical ranges on the following topics:

1. finite-dimensional compressions of shift;
2. companion matrices;
3. nilpotent operators;
4. quadratic operators;
5. nonnegative matrices;
6. Aluthge transform of operator;
7. the boundary of a numerical range;
8. higher-rank numerical ranges.

2-Spectral Sets and Similarity Transformations with Condition Number 2

Anne Greenbaum
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Abstract

Let A be an n by n matrix and let $W(A)$ denote its numerical range:

$$W(A) = \{\langle Aq, q\rangle : \|q\| = 1\}.$$

In *Bounds for analytical functions of matrices*, Integr. Equ. Oper. Theory 48 (2004), pp. 461-477, M. Crouzeix made the interesting conjecture that $W(A)$ is a 2-spectral set for A; that is, for any polynomial p,

$$\|p(A)\| \leq 2\|p\|_{W(A)},$$

where $\|\cdot\|$ denotes the operator 2-norm ($\|p(A)\| = \sup_{\|v\|_2=1} \|p(A)v\|_2$) and $\|\cdot\|_{W(A)}$ denotes the $\mathcal{L}^\infty$-norm on $W(A)$ ($\|p\|_{W(A)} = \sup_{z\in W(A)} |p(z)|$). An equivalent conjecture is that if g is a bijective holomorphic mapping from $W(A)$ onto the unit disk $\mathcal{D}$, then $\mathcal{D}$ is a 2-spectral set for $g(A)$; that is, for any polynomial p, $\|p(g(A))\| \leq 2\|p\|_{\mathcal{D}} \equiv 2\|p\circ g\|_{W(A)}$. This result would follow if it could be shown that $g(A)$ is similar to a contraction via a similarity transformation with condition number at most 2; that is, if $g(A)$ can be written in the form XCX^{-1}, where $\|C\| \leq 1$ and $\kappa(X) \equiv \|X\|\cdot\|X^{-1}\| \leq 2$, since von Neumann's inequality would then imply $\|p(g(A))\| = \|Xp(C)X^{-1}\| \leq \kappa(X)\|p(C)\| \leq 2\|p\|_{\mathcal{D}}$.
Numerical experiments suggest that while the numerical radius of $g(A)$ is usually greater than 1, if one looks for the matrix C of smallest norm satisfying $g(A) = XCX^{-1}$ for some X with $\kappa(X) \leq 2$, then the norm of C is indeed less than or equal to 1. We discuss some numerical observations and possible approaches to proving this.

Compressions of normal matrices

John Holbrook
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Abstract

There has been longstanding interest in the problem of characterizing normal compressions of normal matrices. Indeed, the Hermitian case is completely solved by the Cauchy interlacing theorem, and its converse (due to Fan and Pall). More recently, the theory of higher–rank numerical ranges has included the solution to the case of scalar compressions. Here we take some steps towards a similar treatment of the general case. We develop some natural necessary conditions on the eigenvalues as well as some convenient sufficient conditions, showing by a study of the 2x2 compressions of 4x4 normals that the necessary conditions are not sufficient. We also give a new proof of the Choi–Kribs–Zyczkowski conjecture for 2x2 compressions by means of a powerful extension of that result. The CKZ conjecture (theorem) for 2x2 compressions says that diag(a,a) is a compression of normal N if a lies in the intersection L of the Ck, where Ck denotes the convex hull of the eigenvalues of N with the kth eigenvalue omitted. We show that in fact diag(a,b) is a compression whenever a,b both lie in L. This talk is based on joint work with Nishan Mudalige and Rajesh Pereira.

Supports and regularity for measures in a free additive convolution semigroup

Hao-Wei Huang
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Abstract

Let μ be a Borel probability measure on $\mathbb{R}$ and $t > 1$. In this talk, complete descriptions of the supports of the free additive convolution power $\mu^{\boxplus t}$ will be given. More precisely, I will briefly introduce some important theorems, such as free central limit theorem, in free probability theory and then explain how the supports of the measures in the semigroup $\{\mu^{\boxplus t} : t > 1\}$ vary when t increases. Moreover, motivated by free central limit theorem I will give equivalent conditions so that the measure $\mu^{\boxplus t}$ has only one component in the support for sufficiently large t. An example of μ such that there are infinitely many components in the support of $\mu^{\boxplus t}$ for all $t > 1$ will be given as well.

Isometries of real and complex Hilbert C*-modules

Ming-Hsiu Hsu
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Abstract

Let A and B be real or complex C*-algebras. Let V and W be real or complex (right) Hilbert C*-modules over A and B, respectively. Let T be a bounded linear bijective map from V onto W. We show the following statements are equivalent.

(a) T is a unitary operator and a module map, i.e., there is a $*$-isomorphism $\alpha : A \to B$ such that

$$\langle Tx, Ty\rangle = \alpha(\langle x, y\rangle) \quad \text{and} \quad T(xa) = (Tx)\alpha(a),$$

for all x, y in V and a in A.

(b) T preserves TRO products, i.e.,

$$T(x\langle y, z\rangle) = Tx\langle Ty, Tz\rangle, \quad \forall x, y, z \in V;$$

(c) T is a 2-isometry;

(d) T is a complete isometry.

Moreover, if A and B are commutative, then these statements are hold automatically when T is a isometry. On the other hand, If V and W are complex Hilbert C*-modules over complex C*-algebras, then T is unitary if and only if it is a module map.

This talk is the defense of his PhD thesis of the speaker. Every guest is welcome to participate into the oral exam, and to raise questions and comments in the due course. But all but the members of the exam committee will be asked to leave the room after the talk during the discussion of the exam result.

Duality of Entanglement Norms

Nathaniel Johnston
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Abstract

We consider four norms on tensor product spaces and demonstrate duality relationships between them. We show that the product numerical radius is dual to the robustness of entanglement, and we similarly show that the $S(1)$-norm of "N. Johnston and D. W. Kribs. A Family of Norms With Applications in Quantum Information Theory. Journal of Mathematical Physics, 51:082202, 2010" is dual to the cross norm of "O. Rudolph. A separability criterion for density operators. J. Phys. A: Math. Gen., 33:3951-3955, 2000". We also investigate consequences of this duality.

Private quantum codes: introduction and connections with quantum error correction

David Kribs
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Abstract

This is an introductory and speculative talk on a topic in quantum information science that I've recently been investigating. Private quantum codes arise in quantum cryptography and quantum key distribution. In some sense they are complementary to quantum error correcting codes (and in important special cases this complementarity is made precise by the Stinespring dilation theorem), which themselves have motivated considerable recent work in the study of numerical ranges. However, whereas the theory of quantum error correction is very well developed mathematically, the same cannot be said for private quantum codes.

My acquaintance with Professor Pei-Yuan Wu

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Overview on the Relationship Between Multiplier Operators and Invariant Operators

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Abstract

Let X and Y be Banach spaces, and A be a commutative Banach algebra. Suppose that X and Y are Banach A-module. The space of A-module homomorphisms from X to Y, denoted by $\mathrm{Hom}_A(X, Y)$, is called the space of multipliers from X to Y.
Let G be a locally compact Abelian group, $E(G, X)$ and $F(G, Y)$ be Banach valued spaces on G to X and Y, respectively. If a bounded linear operator $T : E(G, X) \longrightarrow F(G, Y)$ is translation invariant, we will characterize the relationship between the multiplier operators and the invariant operators in this talk.
When $A = L^1(G)$, the multiplier space and the invariant operator space are the same as $M(G)$.

(1) $\mathrm{Hom}_{L^1(G)}(L^1(G), L^1(G)) = (L^1(G), L^1(G)) \cong M(G)$.

(2) $\mathrm{Hom}_{L^1(G)}(L^1(G), F(G)) \cong (L^1(G), F(G)) \cong F(G)$,
if $F(G) = L^p(G)$, $1 \leq p < \infty$ or $F(G) = C_0(G)$.

Moreover, in Banach space valued function defined on G, we can characterize the relation as:

(3) $\mathrm{Hom}_{L^1(G,A)}(L^1(G, A), L^1(G, X)) \cong M(G, X)$.

(4) $\mathrm{Hom}_{L^1(G,A)}(L^1(G, A), L^p(G, X)) \cong L^p(G, X)$. for $1 < p < \infty$.

(5) and others.

Linear and nonlinear disjointness preserving operators on function spaces

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Abstract

We suggest a definition of "biseparating" for a nonlinear operator acting between (vector-valued) function spaces. Then we study representation of biseparating maps acting between spaces of vector-valued differentiable functions. Specializing to linear maps, we recover results of J. Araujo on biseparating maps on $C^p(X, E)$, $p < \infty$. We also obtain some results for the case $p = \infty$.

Optimizing quadratic forms of adjacency matrices, and numerical radii of weighted shifts

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Abstract

We consider optimal arrangement (labeling) of given numbers $w_1 \geq \cdots \geq w_{n-1}$ as weights w_{ij} for the edges e_{ij} of a tree graph with n vertices so that the corresponding quadratic form

$$\sum w_{ij} x_i x_j$$

is maximum/minimum for any nonnegative vector $x = (x_1, \ldots, x_n)$ with entries arranging in descending order. The results are used to determine the optimal arrangement of weights to the unspecified entries of certain operators that give the maximum or minimum numerical radius.

Numerical range associated with a closed orbit under a central force

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Abstract

A closed orbit C under a central force is realized as the dual curve of the boundary generating curve of the numerical range of a matrix $A = H + iK$. In other words, C is expressed as $\det(I_n + xH + yK) = 0$ for some hermitian matrices H, K. This talk is based on joint works with Prof. Mao-Ting Chien of Soochow University.

A Murray-von Neumann type classification of C^*-algebras

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Abstract

We define type $\mathfrak{A}$, type $\mathfrak{B}$, type $\mathfrak{C}$ as well as C^*-semi-finite C^*-algebras. It is shown that a von Neumann algebra is a type $\mathfrak{A}$, type $\mathfrak{B}$, type $\mathfrak{C}$ or C^*-semi-finite C^*-algebra if and only if it is, respectively, a type I, type II, type III or semi-finite von Neumann algebra. Any type I C^*-algebra is of type $\mathfrak{A}$ (actually, type $\mathfrak{A}$ coincides with the discreteness as defined by Peligrad and Zsidó), and any type II C^*-algebra (as defined by Cuntz and Pedersen) is of type $\mathfrak{B}$. Moreover, any type $\mathfrak{C}$ C^*-algebra is of type III (in the sense of Cuntz and Pedersen). Conversely, any purely infinite C^*-algebra (in the sense of Kirchberg and Rørdam) with real rank zero is of type $\mathfrak{C}$, and any separable purely infinite C^*-algebra with stable rank one is also of type $\mathfrak{C}$. We also prove that type $\mathfrak{A}$, type $\mathfrak{B}$, type $\mathfrak{C}$ and C^*-semi-finiteness are stable under taking hereditary C^*-subalgebras, multiplier algebras and strong Morita equivalence. Furthermore, any C^*-algebra A contains a largest type $\mathfrak{A}$ closed ideal $J_{\mathfrak{A}}$, a largest type $\mathfrak{B}$ closed ideal $J_{\mathfrak{B}}$, a largest type $\mathfrak{C}$ closed ideal $J_{\mathfrak{C}}$ as well as a largest C^*-semi-finite closed ideal $J_{\mathfrak{sf}}$. Among them, we have $J_{\mathfrak{A}} + J_{\mathfrak{B}}$ being an essential ideal of $J_{\mathfrak{sf}}$, and $J_{\mathfrak{A}} + J_{\mathfrak{B}} + J_{\mathfrak{C}}$ being an essential ideal of A. On the other hand, $A/J_{\mathfrak{C}}$ is always C^*-semi-finite, and if A is C^*-semi-finite, then $A/J_{\mathfrak{B}}$ is of type $\mathfrak{A}$. Finally, we show that these results hold if type $\mathfrak{A}$, type

$\mathfrak{B}$, type $\mathfrak{C}$ and C^*-semi-finiteness are replaced by discreteness, type II, type III and semi-finiteness (as defined by Cuntz and Pedersen), respectively. [It is a joint work with Ngai-Ching Wong]

Generalized numerical ranges and quantum error correction

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Abstract

For a noisy quantum channel, a quantum error correcting code exists if and only if the joint higher rank numerical ranges associated with the error operators of the channel is non-empty. In this talk, we discuss the geometric properties of the joint higher rank numerical ranges and their implications to quantum computing. It is shown that if the dimension of the underlying Hilbert space of the quantum states is sufficiently large, the joint higher rank numerical range of operators is always star-shaped and contains a non-empty convex subset. In case the operators are infinite dimensional, the joint infinite rank numerical range of the operators is a convex set lying in the star center of all joint higher rank numerical ranges, and is closely related to the joint essential numerical ranges of the operators. In addition, equivalent formulations of the join infinite rank numerical range are obtained. As by products, previous results on essential numerical range of operators are extended.

The numerical range of positive operators

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Abstract

Recently, the numerical range of positive matrices was studied. The authors obtained Perron-Frobenius type results for the numerical range. Many of the tools used there are also available for operators on infinite dimensional spaces. In this talk we try to generalise those results to the numerical range of positive operators on Hilbert lattices.

GMRES and polynomial numerical hulls of matrices

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Abstract

Remarks on the numerical range of banded biperiodic Toeplitz operators: theory and computer generation

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Abstract

Let M_n be the algebra of $n \times n$ complex matrices. A matrix $T_n = (t_{kj}) \in M_n$ is said to be a *biperiodic Toeplitz* matrix if $t_{k,j} := a_{k-j}$, for k odd, and $t_{k,j} := b_{k-j}$, for k even, $k, j = 1, \ldots, n$. If there exists an integer $m \in \mathbb{N}$, $m < n$, such that $a_{k-j} = 0$ and $b_{k-j} = 0$, for $|k-j| > m$, $k, j = 1, \ldots, n$, then T_n is said to be a *banded biperiodic Toeplitz matrix* with *bandwidth* $2m + 1$. Let H^2 be the Hardy space. Any infinite banded biperiodic Toeplitz matrix can be identified with an operator T acting on the $H^2 \times H^2$ space. In this talk we prove that the boundary of the numerical range, $W(T)$, of an infinite banded biperiodic Toeplitz operator coincides with the boundary of the convex-hull of a family of 2×2 matrices. As a consequence of this characterization, a Matlab program that accurately exhibits $W(T)$ is presented. Furthermore, the parametric equations of the boundary generating curves of $W(T)$ are deduced and the numerical range of biperiodic tridiagonal Toeplitz operators is study, identifying a class with an elliptical numerical range. These abstract results are illustrated by several examples. This talk is based on a joint work with Professors Natália Bebiano and J. P. da Providência from the University of Coimbra, Portugal.

Keywords: Numerical range, boundary generating curves, banded biperiodic Toeplitz operators, tridiagonal operators.

Construction of Brain Computing Machines

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Abstract

We wish to introduce a brain computing machine which can be described formally as a 4-tuple, BCM $= (\mathcal{N}, \mathcal{L}, \Pi, \tau)$, where $\mathcal{N}$ is an evolutionary neural network (with random connections), $\mathcal{L}$ is an adaptive plan which determines what plasticity operator is to be applied to modify network structure of $\mathcal{N}$, Π is an aggregation for merging two sequences of vectors of neuronal active states, and $\tau \geq 0$ is a time controller for descrambling outputs of neuronal activity patterns. The outputs of BCM are essentially characterized by the afference of sequential excitatory inputs, the plasticity of neural connections, and the dynamics of nonlinear dynamical systems of coupled neurons. Thus BCM formulates a complex, mathematical system with a highly plastic feature. Two plasticity operators are derived from the measure of synchronous neural activity and the measure of self-sustaining neural activity, respectively. The plasticity operators lead to activity-dependent changes in neural

connections, which meet the neurophysiological postulate of Hebbian synaptic plasticity. With the plasticity operators, we reveal a process of circulation breaking in neural network dynamics: the occurrence of a loop of neuronal active states leads to an activity-dependent change in neural connections, which feeds back to reinforce neurons to tend to break the circulation of neuronal active states in this loop. Circulation breaking in BCM aids in shaping the spatiotemporal patterns of neuronal activity dynamically and culminating in the construct of diverse input-output pairing adaptively. A schematic design of the brain-BCM interface is proposed for the transformation of cortical ensemble plasticity into the action of plasticity operators on BCM.
This is a joint work of Mau-Hsiang Shih and Feng-Sheng Tsai.

The (p,k) matricial ranges and operator quantum error correction

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Abstract

Motivated by operator quantum error correction, we consider the (p,k) matricial range of an $n \times n$ matrix A, which is the set of $p \times p$ complex matrices B such that $X^*AX = B \otimes I_k$ for some $n \times pk$ matrix X with $X^*X = I_{pk}$. In this talk, basic properties/results and open problems on the (p,k) matricial range will be presented.

This talk is based on a joint work with Y.T. Poon (Iowa State University) and C.K. Li (College of William & Mary).

Linear Operators in Nonlinear Analysis and Applications

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Abstract

In this talk, we first study nonlinear analytic methods for linear contractive mappings in Banach spaces and then we obtain some new weak and strong convergence theorems for linear or nonlinear operators in Banach spaces.

Connectedness, Hessian and generalized numerical range

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Abstract

A brief survey on some generalized numerical range associated with a semisimple Lie algebra is given. We give another proof of the convexity of a generalized numerical range associated with a compact Lie group via a connectedness result of Atiyah and a Hessian index result of Duistermaat, Kolk and Varadarajan.
It is a joint work with Xuhua Liu of Auburn University

Prof. Wu's Journey Through Numerical Ranges (Part III)

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Abstract

In this talk, I will introduce the academic contribution of Prof. Pei Yuan Wu about numerical ranges on the following topics:

1. Weighted shift matrices;

2. Numerical radius inequality for contractions.

Fields of values for matrix factorizations and inner $O(n^2)$ approximations of the field of values

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Abstract

This talk has three separate subjects on the field of values of a matrix:

The **first part** investigates relations between the field of values of a matrix A and the fields of values for standard matrix factorizations of A, such as LR, QR, polar etc and we formulate two conjectures.

The **second part** revisits earlier ideas of Marcus and Pesce from 1987 of generating matrix fields of values via compressions and ellipse point evaluations. This approach can theoretically cut the cost of drawing the FOV boundary curve of a matrix $A_{n,n}$ by a factor of n.
But there are difficulties. (joint work with Haley Steger, MA Auburn, 2012)

Finally we show how to reduce the FOV boundary curve plotting cost by around 80 % by using few eigenanalyses in conjunction with 2 by 2 compression ellipses of A instead of only eigenanalyses.

On the numerical range of the weighted shift operators

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Abstract

In this paper we obtain exact formula for $\det(tI_n - (Q_n + Q_n^*))$. Also we find numerical ranges of Q_n and H_n for $n = 3, 4$ in a simple way.

Prof. Wu's Journey Through Numerical Ranges (Part II)

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Abstract

In this talk, I will introduce the academic contribution of Prof. Pei Yuan Wu about numerical ranges on the following topics:

1. Crawford numbers of powers of a matrix;
2. Numerical ranges of weighted shifts;
3. Diagonals and numerical ranges of weighted shift matrices.

Preservers on the Lipschitz functions

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Abstract

Let X, Y be realcompact metric spaces and E, F be Banach spaces. A linear bijection T between the local Lipschitz function spaces $Lip_{loc}(X, E)$ and $Lip_{loc}(Y, F)$ is said to be a *zero-set containments preserver* if

$$z(f) \subset z(g) \Longleftrightarrow z(Tf) \subset z(Tg)$$

for all $f, g \in Lip_{loc}(X, E)$. We prove that the zero-set containments preservers are weighted composition operators $(Tf)(y) = J_y(f(\tau(y)))$, where $J_y : E \to F$ is a linear bijection and $\tau : Y \to X$ is a homeomorphism. Moreover, when the zero-set containments preservers are defined in other Lipschitz functions, we can derive more properties of the homeomorphism τ.

Diagonals and numerical ranges of finite matrices

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Abstract

In this talk, we give a brief survey of our recent works, jointly with H.-L. Gau and K.-Z. Wang, on the relation between the diagonal entries of a compression and the numerical range of a finite matrix. To be more precise, we consider the maximum size of a compression of an n-by-n (n at least 2) matrix A for which all its diagonal entries are in the boundary of the numerical range $W(A)$ of A. Call this number $k(A)$. If A is the n-by-n nilpotent Jordan block, then it's not too difficult to show that $k(A)$ equals the ceiling of $n/2$. We then generalize this fact to the more general S_n-matrices. Another generalization concerns the class of weighted shift matrices. Here the situation is more interesting. We show that, in this case, $k(A)$ can be any integer between 2 and n, and we characterize those A with $k(A)$ equal to n.

Numerical range in complex analysis

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Abstract

In this joint work with Simeon Reich and David Shoikhet, we estimate the deviation of a holomorphic mapping in a Banach space from its linear approximation, the Fréchet derivative at a fixed point, in terms of their numerical ranges. This leads to several constructions of holomorphic retractions onto the fixed point set, and a generalization of Cartan's uniqueness theorem as a characterization of linearity. A number of examples and open questions will be mentioned.

Some inequalities of majorization type

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Abstract

This talk is concerned with matrix inequalities of majorization type. We show some basic majorization inequalities of vectors then apply them to derive matrix inequalities.

Numerical shadow: a probability measure supported by the numerical range

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Abstract

For any operator X acting on an N-dimensional Hilbert space H_N we introduce its numerical shadow, which is a probability measure on the complex plane supported by the numerical range of X. The shadow of X at point z is defined as the probability that the inner product (Xu, u) is equal to z, where u stands for a normalized N-dimensional random complex vector. In the case of $N = 2$ the numerical shadow of a non-normal operator can be interpreted as a shadow of a hollow sphere projected on a plane. A similar interpretation is provided also for higher dimensions. For a hermitian X its numerical shadow forms a probability distribution on the real axis which is shown to be a one dimensional spline.

The notions of numerical range and numerical shadow can be extended for operators acting on a Hilbert space with a tensor product structure. Restricting the set of pure states to the set of product states or maximally entangled states we introduce restricted numerical range and restricted numerical shadow of an operator. Analyzing restricted shadows of operators of a fixed size $N_A \times N_B$ we analyze the geometry of sets of separable and maximally entangled states of composite quantum system.