

## Speech at the banquet of the 11th Workshop on Numerical Ranges and Numerical Radii

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I will start by thanking the host and organizer of this workshop, Prof. Ngai-Ching Wong (黃毅青), for all the works he has done for the past year or so to make this conference so successful. Over the years, he has been an energetic and tireless organizer for various conferences here in National Sun Yat-Sen University (國立中山大學) at Kaohsiung (高雄市). I still remember vividly that, 12 years ago, he organized an international conference to celebrate the dawning of our new millenium, which, with more than 200 participants, is even bigger than the present one. Let's give him a big round of applause to show our appreciations.

There are many other members of our numerical range group who are more deserving to be honored in this way. We have here our senior members, Prof. Fiedler and Prof. Ando. Even into their 80s, they are still carrying on their researches tirelessly as witness their reports in this workshop. They have certainly set up good examples for all of us to follow up. Then Prof. Man-Duen Choi (蔡文端) here is also much more deserving than I am. Many of us are doing our researches in a more-or-less routine manner, but not him. He has always been very clever, much like an elfin (according to the late Domingo Herrero). He looks at things into their innermost essence and devises ingeneous ways to tackle the problems at hand. This I speak from my own experiences while collaborating with him. We have been working together on a 20-plus-year project. The first edition of our joint paper was written up 20 years ago. He is never satisfied with the results and the proofs therein. Over the years, he invented various special

techniques and tricks to have them modified, simplified and generalized. These are typically his trademark; nobody else can have ever thought of them. The resulting latest edition is so completely different from the first one, almost beyond recognition. He has had all my admiration. Another person here we should thank to is Prof. Chi-Kwong Li (李志光). He is one of the initiators of this series of WONRA. Over the years, he has been so productive, not only in numerical range but in all branches of matrix theory. He is quick-minded, full of new ideas, which he dispenses constantly in all occasions he happens to be present, and is always ready to spread the gospel. He is certainly the main reason that the topic of numerical range is so much alive today. In the future, he will surely be honored either in our workshop or in other conferences.

I started my foray into this area about 15 years ago. As you might have known, I have had my Ph.D. from Indiana University back in 1975, working under the guidance of John B. Conway on the dilation theory of Sz.-Nagy and Foias. After my graduation, I came back to Taiwan working in National Chiao Tung University (國立交通大學), thanks to the good advice of my college classmate,

Prof. Tsang-Hai Kuo (郭滄海), and continued my work on this subject for about 10 years. Then I gradually shifted to the general operator theory, working on problems like characterizing the operators which can be expressed as the product of finitely many ones from a certain given class. The first class I considered is that of normal operators. If the space on which the operators act is finite dimensional, then the answer is trivial: every operator is the product of two normal operators via its polar decomposition. The real interest comes from the infinite-dimensional case, in which case, a characterization can be given and we only need 3 normal operators for the product. Then I also expanded to problems on

sums or linear combinations of special operators. This went on for another 10 years.

My interest in numerical range was first aroused in January of 1994 on the occasion of the Amer. Math. Soc. annual meeting in Cincinnati, Ohio. I gave a talk, based on a paper (Diagonal operators: dilation, sum and product, *Acta Sci. Math. (Szeged)*, 57 (1993), 125-138) with Che Kao Fong (方資求) on dilating an arbitrary operator on a Hilbert space to a diagonal one. This we did with an operator-matrix argument. After the talk, Douglas Farenick came up to me, offering another proof by using the result of Boris Mirman (1968) on the dilation of an operator  $A$  with numerical range contained in a triangular region formed by the points  $a$ ,  $b$  and  $c$  to the direct sum of infinitely many copies of the 3-by-3 diagonal operator  $B$  with diagonals  $a$ ,  $b$  and  $c$ . Not only is the proof much simpler but the conclusion is stronger. My initial impression is then upended 3 years later when I had the fortune to supervise the Ph.D. work of Hwa-Long Gau (高華隆). He obtained some excellent results on numerical range in his dissertation of 1998. Since then, he has been driving weekly from Chungli (中壢), where his base of National Central University (國立中央大學) is located, to Hsinchu (新竹) every Wednesday afternoon to discuss with me, which leads to many of our joint works. In 2009, he submitted a paper dedicated to my 65th birthday (Numerical ranges of reducible companion matrices, *Linear Algebra Appl.*, 432 (2010), 1310–1321). I protested that by then I was not that old. For the past 3 or 4 years, I also have Kuo-Zhong Wang (王國仲) to work with, first as my post-doc and then my colleague. Then I have my present post-doc Chi-Tung Chang (張其棟) and my former Ph.D.

student Ming-Cheng Tsai (蔡明誠). Technically, they are all very competent researchers, much stronger than I am. Without them, I certainly won't be able to come this far. I thank them all.

I would also like to thank all the members of our numerical range community. You have all been very supportive. I enjoy your company very much. Some of you may have come to Taiwan for the first time. Please enjoy the hospitality of our host and have a pleasant stay here for the duration of this workshop.

The study of numerical range started almost 100 years ago with the Toeplitz-Hausdorff convexity result in 1918-19. Over the years, it has developed into a respectable research topic, not only with its relation to the inner structure of operators or matrices, but also with its connections with other subjects. A notable example of the latter is its recently-discovered connections with the quantum computing theory. Let's just hope that, with our collective efforts, it will continue to flourish for another 100 years.

(The 11th Workshop on Numerical Ranges and Numerical Radii was held during July 9-12, 2012 at National Sun Yat-Sen University in Kaohsiung, Taiwan, and the banquet in the evening of July 11 at the second floor of Uni-Resort Hotel (西子灣海景商務飯店)).