

雙週一題網路數學問題徵答 105 年度第 1 學期

主辦單位：中山大學應用數學系
補助單位：教育部暨中山大學研究發展處

第四題： 105.11.4 公佈，105.11.18 中午 12 點截止

A gardener plants three maple trees, four oaks, and five birch trees in a row. He plants them in random order, each arrangement being equally likely. Let $\frac{m}{n}$ in lowest terms be the probability that no two birch trees are next to one another. Find $m+n$.

一位園丁種植了三棵楓樹，四棵橡樹以及五棵白樺樹成一列。他隨機順序下種植，每一棵樹被種植的機率都相同。試求沒有兩棵白樺樹相鄰的機率。

解答： First off, notice that there are $\binom{7}{4} = 35$ ways of arranging just the maple and oak trees while ignoring the birch trees.

The five birch trees must now be placed amongst the seven previous trees. We can think of these trees as 7 dividers of 8 slots that the birch trees can go in, making $\binom{8}{5} = 56$ different ways to arrange this. Thus in total, there are $35 \times 56 = 1960$ such arrangements.

There are a total of $\frac{(3+4+5)!}{3!4!5!} = 27720$ ways of arranging the trees, so the requested probability is $\frac{1960}{27720} = \frac{7}{99}$, and $m+n = 106$.

在不考慮白樺樹的情況下，有 $\binom{7}{4} = 35$ 的方法數去排列楓樹與橡樹，試著將五棵白樺樹，安插入 7 棵樹形成的 8 個空位，有 $\binom{8}{5} = 56$ 種不同的方法放置。因此共有 $35 \times 56 = 1960$ 種排列方法。所有樹隨意排列共有 $\frac{(3+4+5)!}{3!4!5!} = 27720$ 方法，所以所求機率為 $\frac{1960}{27720} = \frac{7}{99}$ 。 □

答案請寄至 - 高雄市中山大學應數系圖書館的『雙週一題』信箱，或傳真 07-5253809，或利用電子郵件信箱 nsysu.problem@gmail.com (主旨為「105 年秋季第 X 題解答」)。解答上請註明姓名、校名、校址縣市、系所、年級、班級、學號和 E-mail。