A Study on Factorial Designs with Blocks Influence and Inspection Plan for Radiated Emission Testing of Information Technology Equipment

By

Kam-Fai Wong

Department of Applied Mathematics
National Sun Yat-sen University
Kaohsiung, Taiwan, 804, R.O.C.

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Abstract

Draper and Guttman (1997) show that for basic 2^{k-p} designs, $p \geq 0$, $k-p$ replicates of blocks designs of size two are needed to estimate all the usual (estimable) effects. In Chapter 1, we provide an algebraic formal proof for the two-level blocks designs results and present results applicable to the general case; that is, for the case of s^k factorial ($p = 0$) or s^{k-p} fractional factorial ($p > 0$) designs in s^b blocks, where $0 < b < k-p$, at least $\left\lceil \frac{k-p}{k-p-b} \right\rceil$ replicates are needed to clear up all possible effects. Through the theoretical development presented in this work, it can provide a clearer view on why those results would hold. We will also discuss the estimation equations given in Draper and Guttman (1997).

In Chapter 2, we present a methodology for analyzing the variability of the radiated emission testings of electronic, telecommunication and information technology equipment based on a modified analysis of variance (ANOVA), with polynomial regression analysis. In our study, three electronic products; modem, monitor and notebook bought from the market are tested. Through the experiment, we show that the international standard fails to provide a methodology which gives control limits for EMC when the electronic products in question are produced. We feel that an improved EMC control procedure presented here can better meet the needs of radiated emission control.

Keywords and phrases: Partial confounding, defining sets, estimation equations, analysis of variances, polynomial regression analyses, electromagnetic compatability (EMC).

Chapter 1

s^{k-p} fractional factorial designs in s^b blocks

1.1 Introduction

In a 2^k factorial experiment, there are many situations in which it is impossible to perform all of the runs under homogeneous conditions. For example, a single batch of raw material might not be large enough to make all of the required runs, or a machine can not make all of the required runs in a day, and so on. Confounding is a design technique for arranging a complete factorial experiment in blocks, where the block size is smaller than the number of all possible treatment combinations.

If resources are sufficient to allow the replication of designs confounded with certain effects, it is generally better to use a slightly different method of designing the blocks in each replicate. This approach consists of confounding a different effect in each replicate so that some information on all effects can be obtained. Such a procedure is called partial confounding, which are covered in many textbooks such as Cochran and Cox (1957), Box, Hunter and Hunter (1978), Montgomery (1997) and Wu and Hamada (2000).

However, if we would like to clear up all possible effects in a s^k factorial design in s^b blocks (denote as s_b^{k-p} blocks design), where $0 < b < k$, how many replicates are needed through the technique of partial confounding? Draper and Guttman (1997) discusses the case when two-level designs are split into blocks of size two. Their results state that for basic 2^{k-p} designs, $p \geq 0$, $k-p$ replicates of blocks designs of size two are needed to estimate all the usual (estimable) effects. In Section 1.2.1, we will provide an algebraic formal proof for the 2-level blocks designs result, and present results applicable to the general case, that is, for the case of s^k factorial or $s^{k-p}, p > 0$ fractional factorial design in s^b blocks, $0 < b < k-p$, where s is a prime number in Section 1.2.2 and s is not a prime number in Section 1.2.3 respectively. Through the theoretical development presented in this work, we can provide a clearer view on why those results would hold.

Usually in a 2^k factorial design, it is easy to estimate the mean effects which do not involve block influence. In Draper and Guttman (1997) the estimation equations are employed to find estimates of the mean effects (i.e. $\hat{a}, \hat{b}, \hat{ab}$), so that they are free of block effects, in a 2^k_b blocks design, where all the main effects (i.e. $\hat{A}, \hat{B}$) and interactions (i.e. $\hat{AB}$) free from block effect are estimated first. It is a quite natural and reasonable approach for a 2^k_b blocks design. But from the view of partial confounding, we can see that in Draper and Guttman (1997), the full information has not been used while estimating the interaction in the example given there. We will discuss this in Section 1.3, and suggest to use the interaction estimate with full information in the estimation equations, and so on for other cases through the concept of partial confounding.

1.2 The minimum number of replicates needed to clear up all the effects

To begin with, in the following, let s^k_b blocks or $s^{k-p}_b, p > 0$ fractional blocks design be the s^k factorial or $s^{k-p}, p > 0$ fractional factorial design in s^b blocks, where $0 < b < k-p$; and Ω_{s^k} be the set that contains all the possible effects (or components) with degrees of freedom 1 for the s^k factorial design and identity I. For example,

$\Omega_{3^2} = \{A, A^2, B, B^2, AB, AB^2, A^2B, A^2B^2, I\}$. We provide an algebraic formal proof for the results stated in Draper and Guttman (1997) and its extension to the s-level blocks designs in the following.

1.2.1 Two-level

We begin with 2-level blocks designs. The mathematical method we adopt here is similar to the concept of vector space which can be found in many algebra textbooks. First we introduce the definition of a defining set, which is an abelian group. For the 2-level case, it is the same as the defining contrast subgroup discussed in Bailey (1977). In addition, it actually forms a linear space. Then some terminology developed for vector spaces as in Fraleigh and Beauregard (1995) is applied to the defining set.

Definition 1 *For any $P, Q \in \Omega_{2^k}, PQ$ is the generalized interaction of P and Q, and $P^2 = I$.*

Definition 2 *A nonempty subset S of Ω_{2^k} that is closed under effect generalized interaction is a defining set.*

Definition 3 *Let $E_1, E_2, \cdots, E_m$ be m effects in Ω_{2^k}. The span of these effects is the set of all generalized interaction of them and is denoted by span$(\{E_1, E_2, \cdots, E_m\})$.*

Definition 4 *A set of effects is independent if no effect in the set is the generalized interaction of the other effects in the set.*

Definition 5 *A basis for a defining set is a set of effects which is independent and spans the defining set.*

Definition 6 *The number of effects in a basis of a defining set S is the dimension of S, and is denoted by $dim(S)$.*

Theorem 1 *For any 2_b^k blocks design, there are exactly $2^b - 1$ effects confounded with blocks. Let S be the set that contains the $2^b - 1$ effects and identity I, then S is a defining set.*

Proof: It is known that, in a 2_b^k blocks design, there are exactly $2^b - 1$ effects confounded with blocks. For any two different effects P, Q confounded with blocks, then the effect PQ, a generalized interaction of P and Q, is also confounded with blocks. Thus S is closed under effect generalized interaction, then S is a defining set.

Since the confounding pattern of effects with blocks can be used to determine a block design, by Theorem 1, every defining set S with $dim(S) = b$ is associated with a 2_b^k blocks design.

From the theories about abelian group, the intersection of any two defining sets is still a defining set. Therefore, if S_1 and S_2 represent two replicates to a 2_b^k blocks design, then there are exactly $2^w - 1$ effects still confounded with blocks where $w = dim(S_1 \cap S_2)$. We give a lower bound of $dim(\cap_{i=1}^m S_i)$ in the following.

Theorem 2 *Let $S_i, i = 1, 2, \cdots, m$, be m defining sets representing m 2_b^k blocks designs. Then*

$$dim(\bigcap_{i=1}^{m} S_i) \geq k - (k-b)m.$$

Proof: It is clear that the dimension for any defining set representing a 2_b^k blocks design is equal to b. Then we have

$$dim\,(S_i) = b, \text{ for } i = 1, \cdots, m.$$

Thus the statement is true for $m = 1$. Assume that it is true for $m = r$, then

$$dim(\bigcap_{i=1}^{r} S_i) \geq k - (k-b)r.$$

For $m = r + 1$,

$$\begin{aligned} dim(\bigcap_{i=1}^{r+1} S_i) &= dim(\bigcap_{i=1}^{r} S_i) + dim(S_{r+1}) - dim\left((\bigcap_{i=1}^{r} S_i)\bigcup S_{r+1}\right) \\ &\geq k - (k-b)r + b - dim(\Omega_{2^k}) \\ &= k - (k-b)r - (k-b) \\ &= k - (k-b)(r+1). \end{aligned}$$

Therefore the statement is proved by mathematical induction.

Definition 7 *$\left\lceil \frac{k}{k-b} \right\rceil$ is the smallest integer greater than or equal to $\frac{k}{k-b}$.*

Theorem 3 *At least $\left\lceil \frac{k}{k-b} \right\rceil$ replicates are needed to clear up all possible effects in 2^k_b blocks design.*

Proof: Let $S_i, i = 1, 2, \cdots, m$, be m defining sets representing m 2^k_b blocks designs. If

$$dim(\bigcap_{i=1}^{m} S_i) = w, w \geq 0,$$

it means that there are exactly $2^w - 1$ effects still confounded with blocks. By Theorem 3, we know

$$dim(\bigcap_{i=1}^{m} S_i) \geq k - (k-b)m,$$

so if $k - (k-b)m > 0$, then there are effects still confounded with blocks. Thus we need to find m such that $k - (k-b)m \leq 0$, which implies

$$m \geq \frac{k}{k-b}.$$

Although Theorem 4 gives us only the lower bound of the replicates needed to clear up all possible effects, we know that the lower bound can always be reached. One way of constructing all the $\left\lceil \frac{k}{k-b} \right\rceil$ replicates is to choose the main effects as a basis in each of the defining sets. For example, when $k = 4$ and $b = 3$, at least 4 replicates are needed to clear up all possible effects. We can see that $S_1 = span\{A, B, C\}, S_2 = span\{A, B, D\}, S_3 = span\{A, C, D\}$,and $S_4 = span\{B, C, D\}$ is one of the possible designs that clears up all the effects and reaches the lower bound.

On the other hand, the result of Theorem 3 not only can be used to prove Theorem 4, but also gives us a suggestion on how to construct a 2_b^k blocks design when the number of replicates that can be done is less than $\left\lceil \frac{k}{k-b} \right\rceil$. Similar to the discussion above, a 2_b^k blocks design with m replicates satisfying $dim(\bigcap_{i=1}^m S_i) = k - (k-b)m$ always exists. What we seek are the m replicates such that all the effects in $\bigcap_{i=1}^m S_i$ are of as high-order as possible. This is similar to the idea of fractional factorial design. For example, when $k = 4$ and $b = 3$, if only 3 replicates can be made, then a good choice is to find S_1, S_2 and S_3 such that $\bigcap_{i=1}^3 S_i = \{ABCD\}$. Then $S_1 = span\{ABCD, A, B\}, S_2 = span\{ABCD, C, D\}$, and $S_3 = span\{ABCD, B, D\}$ is one of the choices.

If we regard a 2_b^{k-p} fractional blocks design as a complete 2_b^r blocks design where $r = k - p$, it is clear that $\left\lceil \frac{k-p}{k-p-b} \right\rceil$ replicates are needed to clear up all possible aliases in the 2^{k-p} fractional factorial design. For example, consider a 2^{4-1} fractional factorial design of highest resolution IV, defined by $I = ABCD$. There are 8 treatment combinations; that is, $\{(1), ad, bd, cd, ab, ac, bc, abcd\}$, and 7 possible aliases; that is, $A+BCD, B+ACD, C+BCD, AB+CD, AC+BD, AD+BC$ and $D+ABC$. We can see that if we eliminate the letter d and the effects that contain the letter D, the treatment combinations become $\{(1), a, b, c, ab, ac, bc, abc\}$, and the possible aliases become A, B, C, AB, AC and ABC, which are the treatment combinations and possible main effects and interactions in a 2^3 factorial design. This is an easy and natural extension of the 2_b^k blocks design. Next we consider prime s-level factorial designs in the same manner.

1.2.2 Prime s-level

For any prime s, each of the elements x in Ω_{s^k} generates a cyclic subgroup $< x >$ of Ω_{s^k} with order s. For any two cyclic subgroups $< x_1 >$ and $< x_2 >$, either $< x_1 >=< x_2 >$ or $< x_1 > \bigcap < x_2 >= \{I\}$. This has also been discussed in Bailey (1977). Therefore, any of them can be used to generate a s_1^k blocks design directly. We now replace the generalized interactions stated in 2_b^k blocks design by the following definition.

Definition 8 *For any $P, Q \in \Omega_{s^k}$, P^2 is the generalized interaction of P, PQ is the generalized interaction of P and Q, and $P^s = I$.*

After we modification, it is clear that for any prime s_b^k blocks design, there are exactly $s^b - 1$ effects (or components) confounded with blocks. The set S that contains the $s^b - 1$ effects confounded with blocks and identity I is still a defining set, and the dimension of S is still equal to b. Thus all the theorems stated before can be extended to s_b^k blocks designs.

As an illustration, when $s = 3$,$k = 4$ and $b = 3$, then

$$S = span\{A, B, C\} = \{A^a B^b C^c | 0 \leq a, b, c \leq 2\}$$

is a defining set represent a 3_3^4 blocks design. Similar to $s = 2$, we can see that $S_1 = span\{A, B, C\}, S_2 = span\{A, B, D\}, S_3 = span\{A, C, D\}$,and $S_4 = span\{B, C, D\}$ is one of the 3_3^4 blocks design with 4 replicates, which clears up all possible effects.

1.2.3 The other s-level

Although, for any s which is not a prime number, not all the elements in Ω_{s^k} generates a cyclic subgroup of Ω_{s^k} with order s. This implies that not all the elements in Ω_{s^k} can be used to generate a s_1^k blocks design directly. For example, when $s = 4$ and $k = 2$, if we use the defining contrast $L = 2x_1 + 2x_2 \pmod 4$ to generate a block design (i.e.

$S = span\{A^2B^2\}$), then we can only separate all the treatment combinations into two blocks.

It is clear that, the relation between two cyclic subgroups of Ω_{s^k} is more complicate for non-prime s. For example, when $s = 4$ and $k = 2$, then $< A >= \{A, A^2, A^3, I\}$ and $< AB^2 >= \{AB^2, A^2, A^3B^2, I\}$. We can see that $< A > \bigcap < AB^2 >= \{A^2, I\}$. In this manner, although AB^2 is not a generalized interaction of A, span$\{A, AB^2\}$ still can not represent a 4^k_2 blocks design. Therefore, we must revise the definitions of a defining set and effects independence respectively with some more restrictions on it.

Definition 9 *A nonempty subset S of Ω_{s^k} that is closed under effect generalized interaction and the number of elements in S is equal to $s^b, 0 < b < k$, is a defining set.*

Definition 10 *A set of effects is independent if no effect in the set is the generalized interaction of the other effects in the set and the interaction of two cyclic subgroups which are generated by any two effects in the set is $\{I\}$.*

All the theorems stated in Section 2.1 still hold here for all non-prime s-level blocks design under the revised definition. A minor difference is that $dim(S) = b$ may not hold for some defining set S of s^k_b blocks design when s is not prime. For example, $S = \{A^2, B^2, A^2B^2, I\}$ is a defining set of 4^3_1, but $dim(S) = 2$. However, $dim(S) \geq b$ holds. This will not affect the result of the Theorem 2. We give a corollary summarizing the result.

Corollary 1 *At least $\left\lceil \frac{k}{k-b} \right\rceil$ replicates are needed to clear up all possible effects in a s^k_b blocks design.*

Actually, the definitions stated in this section is also suitable to the prime s-level. This is because of the additional conditions given in definitions 9 and 10 always hold for any prime s-level. Therefore, if we start with definitions 8, 9 and 10 instead of definitions 1,2 and 4, all the theorems can be proved in the general case directly.

1.3 Estimation of mean effects under block influence

In this section, we will discuss the method of estimation equation given in Draper and Guttman (1997). For purpose of illustration, the data in Draper and Guttman (1997) are listed in Table 1.1 as follows.

Table 1.1: Partial confounding in the 2_1^2 blocks design.

Replicate I		Replicate II	
A Confounded		B Confounded	
$(1) = 16$	$a = 44$	$(1) = 38$	$b = 66$
$b = 68$	$ab = 118$	$a = 42$	$ab = 92$
Block 1	Block 2	Block 3	Block 4

We can see that A is confounded in replicate I, and B is confounded in replicate II. In principle, we may use all observations in both replicates to estimate the interaction AB. Only replicate II has been used in Draper and Guttman (1997) to estimate the interaction AB. It seems to be inconsistent with the general way that we are familiar with estimating interaction with partial confounding. Although in this example, coincidentally the same result is obtained. In general, they are not the same.

Usually when there is no block influence, the common way to estimate all the mean effects is by the corresponding averages. After that, all the main effects and interactions are estimated through their contrasts. For more details, see Montgomery (1997, p.315-318). The estimates using the averages are given in the first row of Table 1.2. The estimation equations are employed in Draper and Guttman (1997) to estimate the mean effects (i.e. $\hat{a}, \hat{b}, etc$) of a 2_b^k blocks design, where all the main effects and interactions are estimated first, then the mean effects are estimated afterwards. This reverses the usual order of estimations through the same linear system between the main effects, interactions etc. and those mean effects for treatment combinations. By this method, since the estimation of main effects and interactions are cleared up with blocks, so the estimation of mean effects are also cleared up with blocks. It is a quite natural and reasonable approach for a 2_b^k blocks design. The results using estimation equations approach are given in row two of

Table 1.2. Note that there is a slight difference between the results presented in Draper and Guttman (1997) and that in Table 1.2, since the interaction effect AB is estimated with full information, i.e. using all the data.

Table 1.2: Estimates of effects with and without block influence.

	$\hat{(1)}$	$\hat{a}$	$\hat{b}$	$\hat{ab}$	$\hat{\mu}$	$\hat{A}$	$\hat{B}$	$\hat{AB}$
With Block Influence	27	43	67	105	60.5	27	51	11
Without Block Influence	27	31	79	105	60.5	15	63	11

From this example, it is easy to see why using estimation equations to estimate mean effects would be better than using the average for each mean effect. In Replicate I, the main effect A is confounded with blocks. If we estimate A with the data in Replicate I, then $\hat{A}$, the estimate of A, equals to 39 ($\frac{44+118-16-68}{2} = 39$). However, the $\hat{A}$ got from Replicate II where the main effect A is not confounded with blocks equals to 15 ($\frac{42+92-38-66}{2} = 15$). Therefore, it is clear the block effect is significant in Replicate I. Similarly, the block effect is significant in Replicate II. On the other hand, we can see that value of $(1) = 16$ in Block 1 is less than the one of $(1) = 38$ in Block 3; value of $a = 42$ in Block 3 is near to the one of $a = 44$ in Block 2; value of $ab = 118$ in Block 2 is greater than the one of $ab = 92$ in Block 4; value of $b = 66$ in Block 4 is near to the one of $b = 68$ in Block 1. Intuitively it seems that values in Block 1 and Block 4 are lower than true values, and those in Block 2 and Block 3 are higher than true ones. Hence if we adopt the average of the values of mean effect a in Block 2 and Block 3 to do the estimation for mean effect a, that is $\hat{a} = 43$, the estimation of a would be overestimated; if we adopt the average of the values of mean effect b in Block 1 and Block 4 to do the estimation for mean effect b, that is $\hat{b} = 67$, the estimation of b would be underestimated. The results of mean effects given by the estimation equations are shown in Table 2. It appears that block effects have been cleared up if we employ the method of estimation equations.

1.4 Conclusion

In general, to estimate all the usual effects in a s_b^k blocks ($p = 0$) or fractional blocks ($p > 0$) design, $0 < b < k - p$, $\left\lceil \frac{k-p}{k-p-b} \right\rceil$ replicates are required. If only $\left\lceil \frac{k-p}{k-p-b} \right\rceil - l, 0 < l < \left\lceil \frac{k-p}{k-p-b} \right\rceil$, replicates can be made, then we can always find a design such that there are exactly $(s^l - 1)$ effects (or components) confounded with blocks. Since the replicates can be chosen in many different ways, what we would like to do is to find the replicates that all the effects still confounded with blocks are chosen with as high-order as possible.

On the other hand, through the concept of isomorphism of groups which can be found in many algebra textbooks such as Joseph (1986), some of the mixed level cases can be settled. For example, a 6_b^k blocks design is isomorphic to a $2^k \times 3^k$ factorial design in $2^b \times 3^b$ blocks, thus at least $\left\lceil \frac{k}{k-b} \right\rceil$ replicates are needed to clear up all possible effects in both designs. However, a 18_b^k blocks design is isomorphic to a $2^k \times 9^k$ factorial design in $2^b \times 9^b$ blocks, but may not be isomorphic to a $3^k \times 6^k$ factorial design in $3^b \times 6^b$ blocks. The discussions for mixed level cases involve more corresponding results in algebra and will not be discussed here.

Furthermore, the estimation equations introduced by Draper and Guttman (1997) are effective and useful in clearing up the block influence, but if we want to get more information, the concept of partial confounding should be considered.

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Chapter 2

Inspection plan for radiated emission testing of information technology equipment

2.1 Introduction

As the use of electronic products is growing rapidly in most households in both developed and developing countries, the importance of electromagnetic compatability (EMC) can not be overstated. In short, the EMC of a piece of electronic equipment is its ability to meet relevant international standards. As a result it will not generate excessive radiated emission, which is a disturbance to other electronic equipment. Many electronic product malfunction has been incurred by radiated electromagnetic waves from various sources. It has been acknowledged generally that in different directions and position, the radiated emission measurements would be quite different. In another words, there are large variabilities between the measurements in different positions. It has already been a major concern for the EMC association in establishing a more accurate testing procedure. This work is to study the shortcomings of current control procedures stipulated in CISPR standards and try to propose a inspection plan to better control the radiated emission from the information technology equipment.

Before illustrating the problem we are studying, the standard test procedure is stated first in the following:

1. To adjust the broadband antenna to horizontal polarization.
2. To place the EUT on a non-metallic table of 80 cm above the horizontal metal ground.
3. To adjust the frequencies from 30 MHz to 1000 MHz in increasing orders.
4. To adjust the antenna between 1m and 4m in height and to rotate the EUT table 360 degrees trying to detect a local maximum in disturbance.
5. If the maximum radiated emission exceeds 30dB(uv/m) when frequency is between 30 MHz and 250 MHz or exceed 37dB(uv/m) when frequency is between 250 MHz and 1000 MHz, then the test of the product is a failure and the testing procedure shall be terminated.
6. To adjust the antenna from horizontal to vertical polarization and carry out the test procedure again. The product is acceptable if the two standard limits; 30dB(uv/m) and 37dB(uv/m) are not exceeded.

A flow chart of the standard procedure is presented in Figure 2.1.

According to the standard test procedure, 4 main factors; polarization of the broadband antenna, equipment-under-test table height, antenna height and frequency, are to be tested. However, the international standard fails to provide a methodology which gives control limits for EMC when the electronic products in question are produced. In addition, the inherent variability of the EMC test results is not mentioned. We feel that an improved EMC control procedure presented here can better meet the needs of radiated emission control. Many electronic equipment malfunction currently reported in average households, factories and hospitals can be greatly reduced.

In our study, three electronic products; modem, monitor and notebook bought from the market are tested. We are interested in knowing whether repeated measurements, other levels of antenna height and table height would make a difference on the EMI readings and should be considered in the testing procedure. Therefore, we have included one more level for table height and two more levels for antenna height in this experiment and have tested them with replication. Then the factorial experiment becomes one where the factor A-antenna polarization has two fixed levels: horizontal and vertical, factor B-EUT table height has two fixed levels: 40 cm and 80 cm, factor C-antenna height has 3 different levels: 1 to 4 meters, 4 to 5 meters and 5 to 6 meters, and factor D-frequency has six fixed levels. The six fixed levels for the frequency are chosen where local maximum disturbances (radiated emissions) are observed when polarization is horizontal, table height is 40cm and antenna height is adjusted from 1 to 4 meters. The whole experiment is repeated one more time with the same testing procedure under the same equipment.

Due to the actual practice in collecting data, the level changes of frequency are administered at an increasing order in each test run. That is the EMI are measured while the frequency is set from 30MHz to 1000MHz when the other three factors are fixed. Before analyzing the data, a natural question is whether the EMI measurements are independent and have the same variance with respect to frequency. This will be discussed first in Section 2.2 using MANOVA. In Section 2.3, we will try to determine whether the main effects and interactions between factors have significant influences through analysis of variance (ANOVA). We use ANOVA instead of MANOVA in Section 2.3 because it is easier for the engineers to understand, and the interaction between factor D-frequency and other factors can also be considered. For the purpose of controlling the radiated emission of the electronic equipment, it is required that the maximum radiated emission should not exceed the standard control values. If the maximum response always occurs at either one of the two treatment combinations stated in the standard testing procedure, it is not necessary to test at other treatment combinations even though the effects and interactions of the factors are significant. Thus we would like to know under which treatment combination does the maximum response occur. Since the radiated emissions are recorded at six frequencies only, we try to build the extreme value response curves for each of the treatment combinations based on readings at the six frequencies with one replication, and found the estimates of maximum response in Section 2.4. The curves established here are not exactly the true response curve but one approximates the trend of the extreme values. In Section 2.5, we will give more detailed discussions about the estimation of the response curve and suggest a modified analysis of testing procedure. We plan to test another product which has the same model to ours study and give an inspection plan for the testing agencies in the future.

2.2 Response correlation analysis

There are four factors in the experiment; polarization of the broadband antenna, equipment-under-test table height, antenna height and frequencies. The testing procedure we use is similar to the standard testing procedure stated in Section 2.1. A slight difference to the standard procedure is that we have included one more level for table height and two more levels for antenna height, and only disturbances at six fixed frequencies for each run are recorded, where a run is a part of the experiment when the levels of factor A, B and C are fixed. This is a standard mixed levels factorial design with four factors if the observations are taken with random order. Due to the actual practice in collecting the data, the order of obtaining the observation for the six levels of frequency is not random. Then the responses obtained may be correlated. Therefore, we first check whether the responses are independent and have equal variances with respect to the frequency through the technique of multivariate analysis of variance (MANOVA).

Now record observations at the six frequencies as a 6×1 vector for each run, that is to record the disturbances when the level of factors A, B and C are fixed. Therefore, the observations may be described by the multivariate linear statistical model

$$Y_{ijkn} = \mu + R_n + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \gamma_k + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \varepsilon_{ijkn} \tag{2.1}$$

for $i = 1, 2; j = 1, 2; k = 1, 2, 3$; and $n = 1, 2$, where Y_{ijkn} is the observed 6×1 vector at the six test frequencies, μ is the overall mean effect, R_n is the random effect between replicates, α_i is the effect of the ith level of the factor A, β_j is the effect of the jth level of the factor B, $(\alpha\beta)_{ij}$ is the effect of the interaction between α_i and β_j, γ_k is the effect of the kth level of the factor C and so on. ε_{ijkn} is a random error component where $\{\varepsilon_{ijkn}\}$ are assumed to be $i.i.d$ multivariate normal with mean zero vector and a 6×6 covariance matrix Σ.

Through MANOVA, we obtain the estimates of covariance matrices $\sum$ for the three products respectively and present them in Appendix A1. If the hypothesis of $\sum = \sigma^2 I$ can be assumed, then we may proceed the analysis with the usual univariate ANOVA and investigate the effects of the four factors including frequency. If not, then we will transform the observations so that the covariance matrix is approximately of the form $\sigma^2 I$ for later analysis.

To test the hypothesis that $H_0 : \sum = \sigma^2 I$ vs $H_a : \sum \neq \sigma^2 I$, let

$$u' = -\left(v - \frac{2p^2 + p + 2}{6p}\right) \ln(\frac{p^p \left|S\right|}{(trS)^p})$$

where S is an estimate of the $p \times p$ covariance matrix with degrees of freedom v. Since u' has an approximate χ^2_f-distribution, $f = \frac{1}{2}p(p+1) - 1$, under null hypothesis, we reject the null hypothesis if $u' > \chi^2_{\alpha,f}$. For more details, please see Rencher (1995, p274-275). In our experiment, the critical value $\chi^2_{0.05,20} = 31.41$ and u' equals to $25.99, 70.36$ and 61.77 for modem, monitor and notebook respectively. The null hypothesis is rejected for monitor ($p < 0.001$) and notebook ($p < 0.001$) but accepted for modem ($p \approx 0.17$).

Although not surprisingly, the assumption that the responses at different frequencies are independent and have equal variances is rejected for monitor and notebook, a transformation of the observed vector would still be easier if the assumption of independence still holds. It can be expressed as $\sum = D$ for some diagonal positive definite matrix D. So in the second step, we test the null hypothesis that $H_0 : \sum = D$ for some diagonal positive definite matrix D.

Now let $u^* = -\left[v - \frac{1}{6}(2p+5)\right] \ln|R|$ where R is an $p \times p$ correlation matrix with degrees of freedom v. Under H_0, u^* has an approximate χ^2-distribution with $\frac{1}{2}p(p-1)$ degrees of freedom. Thus we reject H_0 if $u^* > \chi^2_{\frac{1}{2}p(p-1)}$. For more details, please also see Rencher (1995, p290-291). Unfortunately, we have $u^* = 54.34$ ($p < 0.0001$) and $u^* = 32.45$ ($p \approx 0.005$) for monitor and notebook respectively, and the null hypothesis that the observations are independent is still rejected.

Regarding to the above results, we have to transform the data of monitor and notebook before doing the univariate ANOVA. For the purpose of comparison, we still transform the data of modem even though the null hypothesis $H_0 : \sum = \sigma^2 I$ is accepted for modem. For clarity, we first illustrate the method of transformation between the three types of equipment in next section, then present the results of the ANOVA.

2.3 Statistical analysis of the fixed effects model

The method of transformation we adopt here is similar to the concept of weighted least squares which can be found in many regression analysis textbooks. Since any covariance matrix $\sum$ is non-negative, here we assume further to be positive definite, then there exists a positive definite matrix Q such that $\sum = Q^2$. We may transform the observed vector Y to be $Z = Q^{-1}Y$.

After transforming the observations by using the estimates of covariance matrices in Appendix A1, we perform the ANOVA to answer whether the main effects and interactions are significant for the modem, monitor and notebook respectively.

The transformed observations may be described by the univariate linear statistical model

$$Z_{ijkln} = \mu + R'_n + \alpha'_i + \beta'_j + (\alpha\beta)'_{ij} + \gamma'_k + (\alpha\gamma)'_{ik} + (\beta\gamma)'_{jk} + (\alpha\beta\gamma)'_{ijk} + \delta_l \qquad (2.2)$$
$$+(\alpha\delta)_{il} + (\beta\delta)_{jl} + (\alpha\beta\delta)_{ijl} + (\gamma\delta)_{kl} + (\alpha\gamma\delta)_{ikl} + (\beta\gamma\delta)_{jkl} + (\alpha\beta\gamma\delta)_{ijkl} + \varepsilon'_{ijkln}$$

for $i = 1, 2; j = 1, 2; k = 1, 2, 3; l = 1, 2, 3, 4$; and $n = 1, 2$, where Z_{ijkln} is the observed value after transformation and $R'_n, \alpha'_i, \beta'_j, \gamma'_k, \delta_l$ and $(\alpha\beta)'_{ij}$ etc. are the effects of the corresponding factors.

The results of ANOVA are given in Tables 2.1, 2.2 and 2.3 for the modem, monitor and notebook respectively. It can be found that except the replicate R'_n is non-significant for both modem and notebook, the interaction AB, BC and ABC are non-significant for the modem, and the interaction BC is non-significant for the notebook, all the other main effects and interactions are significant for all three products. From the strong significance of the replicate for the monitor, it shows that the stability of the monitor is the worst. This is confirmed with the experience of the engineers.

Furthermore, we can see that the sum of squares of the effects related to D dominates all the other main effects and interactions. For this reason, we also provide the partial R-square to see how much variation in the mean response is associated with the other main effects and interactions when all the effects related to D is already in the model. The results also given in Tables 2.1, 2.2 and 2.3 for the three products respectively.

From the analysis up to now, we have arrived at some conclusions as follows. Through the estimates of the covariance matrices given in Appendix A1, we know that replication is needed to estimate the inherent variability for the equipment. In fact, in many real cases it is not unusual that a prototype failed in the first testing can pass the second or third testing without any change of the prototype's design without much difficultly. This is because the standard testing procedure does not consider the inherent variabilities. For instance, suppose the maximum response of a prototype has the value $\hat{\mu}_{\max} = 37.5$ and $\hat{\sigma}_{\max} = 1.5$. Then the probability that the prototype would pass in each test equals to $P(Z < \frac{37-37.5}{1.2}) = 0.37$. This means that the probability of the prototype would pass at least once with no more than four testings would be equal to 0.84 even though it should be considered to be a failed product.

Table 2.1. Analysis of variance for the modem

Source	DF	Anova SS	Partial R-Square	Mean Square	F Value	Pr > F
R	1	3.19	0.017	3.19	3.30	0.0737
A	1	15.13	0.052	15.13	15.62	0.0002
B	1	25.29	0.137	25.29	26.10	0.0001
C	2	29.20	0.158	14.60	15.07	0.0001
D	5	19036.88		3807.38	3929.48	0.0001
A*B	1	0.39	0.002	0.39	0.41	0.5261
A*C	2	37.29	0.201	18.64	19.24	0.0001
A*D	5	2443.45		488.69	504.36	0.0001
B*C	2	1.47	0.008	0.73	0.76	0.4727
B*D	5	739.26		147.85	152.59	0.0001
C*D	10	85.97		8.60	8.87	0.0001
A*B*C	2	4.47	0.024	2.24	2.31	0.1068
A*B*D	5	296.30		59.32	61.22	0.0001
A*C*D	10	411.88		41.19	42.51	0.0001
B*C*D	10	79.50		7.95	8.20	0.0001
A*B*C*D	10	33.12		3.31	3.42	0.0011
Error	71	68.79		0.97		
Total	143	23311.88				

Table 2.2. Analysis of variance for the monitor

Source	DF	Anova SS	Partial R-Square	Mean Square	F Value	Pr > F
R	1	9.02	0.013	9.02	10.16	0.0021
A	1	289.93	0.420	289.93	326.76	0.0001
B	1	40.94	0.059	40.84	46.14	0.0001
C	2	127.93	0.185	63.96	72.09	0.0001
D	5	209526.43		41905.29	47228.28	0.0001
A*B	1	8.37	0.012	8.37	9.43	0.0030
A*C	2	136.50	0.198	68.25	76.92	0.0001
A*D	5	5543.10		1108.62	1249.44	0.0001
B*C	2	7.30	0.011	3.65	4.11	0.0204
B*D	5	294.23		58.85	66.32	0.0001
C*D	10	688.31		68.83	77.57	0.0001
A*B*C	2	7.40	0.011	3.70	4.17	0.0194
A*B*D	5	82.73		16.55	18.65	0.0001
A*C*D	10	1480.97		148.10	166.91	0.0001
B*C*D	10	193.44		19.34	21.80	0.0001
A*B*C*D	10	344.00		34.40	38.77	0.0001
Error	71	63.00		0.89		0.0001
Total	143	218843.62				

Table 2.3. Analysis of variance for the notebook

Source	DF	Anova SS	Partial R-Square	Mean Square	F Value	Pr > F
R	1	1.74	0.001	1.74	1.76	0.1893
A	1	126.94	0.066	126.94	128.02	0.0001
B	1	143.19	0.075	143.19	144.40	0.0001
C	2	572.14	0.298	286.07	288.50	0.0001
D	5	1003760.50		200752.10	99999.99	0.0001
A*B	1	416.68	0.217	416.68	420.22	0.0001
A*C	2	264.66	0.138	132.33	133.46	0.0001
A*D	5	1896.57		379.31	382.54	0.0001
B*C	2	2.86	0.001	1.43	1.44	0.2434
B*D	5	453.60		90.72	91.49	0.0001
C*D	10	964.01		96.40	97.22	0.0001
A*B*C	2	318.45	0.166	159.22	160.58	0.0001
A*B*D	5	1354.54		270.91	273.21	0.0001
A*C*D	10	1800.33		180.03	181.56	0.0001
B*C*D	10	190.31		19.03	19.19	0.0001
A*B*C*D	10	898.12		89.81	90.57	0.0001
Error	71	70.40		0.99		
Total	143	1013235.03				

In addition, all the products in this tested experiment were bought from the market, they are finished products. However they all failed while using the standard testing procedure. This is a strong evidence that the standard testing procedure is not proper in controlling the radiated emission quality of the products. It is with high probability that a prototype is actually still unqualified even though it has passed the test in inspection. We need a modified testing procedure which not only has a reasonable type I error but also has high testing power to distinguish the unqualified products.

On the other hand, we can see that most of the effects and interactions are significant through the results of ANOVA. Therefore, to prevent excessive electromagnetic interference from appearing in the residential area, manufacturers of electronic products should regularly check radiated emissions of their products with other levels of table height and antenna height. In another manner, other levels of table height and antenna height could be out of consideration only if the maximum response occurs in the present level almost surely.

Since we record the radiated emissions at six frequencies only, without any modification, it is not easy to obtain the exact position of maximum response. We try to build up a extreme values response curve based on the six local maxima and found the estimates of maximum response through the estimated extreme values response curve next.

2.4 Extreme values response curve

Since the effect of frequency is significant, a response curve can be fitted to the data so that the radiated emission may be predicted as a function of the frequency. However, the significant $ABCD$ interaction implies that the radiated emission response to frequency depends on which level of antenna polarization, table height and antenna height it has. Therefore, there would be twelve response curves, one for each combination.

Now we will use weighted least squares to estimate the extreme values response curve.

The statistical model is described as

$$y = \sum_{m=0}^{5} S_m(x_1, x_2, x_3, x_4) x_5^m + \varepsilon \tag{2.3}$$

where

$$S_m(x_1, x_2, x_3, x_4) = \beta_{0,m} + \sum_{i=1}^{4} \beta_{i,m} x_i + \sum_{i<j}^{4} \beta_{i,j,m} x_i x_j + \sum_{k=1}^{2} \beta_{1,2,k,m} x_1 x_2 x_k \tag{2.4}$$

and $\beta_{3,4,m} = 0$ with $m = 0, \ldots, 5$;

$$\begin{aligned}
x_1 &= \begin{cases} 0, & \text{if the antenna polarization is horizontal} \\ 1, & \text{if the antenna polarization is vertical} \end{cases}; \\
x_2 &= \begin{cases} 0, & \text{if the table height is } 40cm \\ 1, & \text{if the table height is } 80cm \end{cases}; \\
x_3 &= \begin{cases} 1, & \text{if the antenna height is } 4\tilde{\ }5m \\ 0, & \text{otherwise} \end{cases}; \\
x_4 &= \begin{cases} 1, & \text{if the antenna height is } 5\tilde{\ }6m \\ 0, & \text{otherwise} \end{cases}; \\
x_5 &= \frac{1}{100}[F - (F_{\max} + F_{\min})/2]
\end{aligned}$$

and F is the frequency(MHz), $F_{\max}$ and $F_{\min}$ are the maximum and minimum frequency recorded respectively, and ε is a random error component.

We can see that model (2.3) is a 5th-order polynomial regression in one variable-frequency when the level of the other three factors are given. The function $S_m(x_1, x_2, x_3, x_4)$ in the model corresponds to all the treatment combinations of the first three factors. Although the factor C-antenna height is separated into x_3 and x_4 since there are three ordinal levels. For example, x_1 is corresponding to main effect A, $x_1 x_2$ is corresponding to interaction AB, $x_1 x_3$ and $x_1 x_4$ are corresponding to interaction AC, and so on. The model is written in this form so that the results of ANOVA can be employed to eliminate some parameters.

Since the interactions AB, BC and ABC are nonsignificant for the modem, thus we assume that $\beta_{1,2,0} = \beta_{2,3,0} = \beta_{2,4,0} = \beta_{1,2,3,0} = \beta_{1,2,4,0} = 0$ for the modem. Similary,$\beta_{2,3,0}$ and $\beta_{2,4,0}$ are assumed to be zero for the notebook.

The results are given in Figures 2.2 through 2.7 for the modem, monitor and notebook respectively in Appendix. Due to the estimate of maximum response curves we have, it is reasonable to say that the curves are the estimate of the trend of local maximum responses at each treatment combination. Recall that the six frequencies we have recorded are selected because the six locally highest disturbances (radiated emissions) are observed when the level of the run is horizontal, $40cm$ and $1\tilde{}4m$. However, it is possible to have different location of frequencies that a local maximum occur even though the level of run is same. In addition, it is possible that locally maximum responses of other runs occur at these six frequencies.

Based on the estimation we have, the maximum responses would occur when the treatment combination is vertical, $80cm$, $4\tilde{}5m$ and 558.40MHz for the modem; vertical, $40cm$, $1\tilde{}4m$ and 81.756MHz for the monitor; and vertical, $40cm$, $5\tilde{}6m$ and 723.07MHz for the notebook. All of them are different from that for the standard treatment combinations.

Evidently, the estimate of maximum response we have is still not exactly correct. This is because in the original experiment, the maximum response has not been estimated at first. Due to the original problems we have, that is whether repeated measurements, other levels of antenna height and table height would make a difference on the EMI reading, in the standard testing procedure with six levels of frequency is enough to answer that question. However, it is clear that only six levels for frequency is not enough to fit the response curve well and test whether a product has met the control limits. To find the real maximum of the response curve, we have to improve our experimental design. We will discuss it in Section 2.5.

2.5 Discussion

From the results of ANOVA in Section 3, we know that the main effect D-frequency dominate all the other effects and interactions. Furthermore, there are always six local maxima responses within the frequency limits tested for each run from the past experiences. This shows that the pattern of the EMI readings at each run can well be approximated by a 12th-order polynomial when the variable is frequency. Then, only six records at each run is not easy for us to fit the response curve very well. Therefore it is clear that readings at more frequency levels are needed. This actually is not difficult to collect more response values in practice.

In the following, we plan to use polynomial model as an approximation to the true function. Through the experience of the practitioners, six local maxima always exist at each run. Since the extra effort and cost is minimum even though all the collected frequencies for each run, we suggest to record all of them first. After that take out eleven frequencies where the six local maxima and five local minima locate in a vector for each run, and find the mean vector by the vector we have recorded. Then we consider the resulted frequencies in the mean vector. Furthermore, the end points, 30MHz and 1000MHz, and the intersection frequency of low- and high- frequencies, 250MHz, should also be considered. Then totally there are fourteen levels for the frequency.

We give the reason of the choice of levels of frequency as follows:

1. We are interested in the maximum response. The best choice is to record the corresponding frequency where the maximum response occurs. However, according to the inherent variability, it is difficult to know exactly the locations of the frequencies where a maximum occurred. So we try to record the frequencies that are near to the location of a local maximum response, it should not be too bad. Under the assumption that the corresponding frequencies where a local maximum exists at similar position for each run, we consider the resulted frequencies in the mean vector.

2. In Section 2.4, some of the estimate of maximum response of each run is not good. This is because we have not described the trend of the true response curve very well. For instance, through the extreme value response curve given in Figure 2.2(d), the estimates

of the trend of the responses between 400MHz and 650MHz is a second-order polynomial, but the trend of the responses should be a fourth-order polynomial. Therefore, we would also like to consider the frequencies where a local minimum occurred.

3. We know that the global maximum of a polynomial on a closed interval occurs either at the local maxima or the end points. Since the specification for low- and high-frequencies is different, the end points for the low-frequency are 30MHz and 250MHz, and the end points for the high-frequency are 250MHz and 1000MHz, we should also consider them.

When the levels of frequency are increased to be of 14 of them, the analysis stated above should be modified accordingly. If we record observations in a vector at each run, then the degrees of freedom for the estimates of covariance matrix in the MANOVA analysis is twelve. It is not enough to estimate the covariance matrix if the dimension of the vector is greater than twelve. Recall that the levels of frequency in the modified analysis of experimental design is fourteen. We suggest to consider low- and high-frequencies separately. This is acceptable since the specification for the low- and high-frequencies are also given in two categories. In this case, we suggest the use of polynomial spline model with one knot to estimate the response curve. This will be discussed in the future work. Furthermore, we plan to test another product with the same model as in ours study and give an inspection plan for the testing agencies in the future.

References II

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Appendix

A1. Estimates of covariance matrices for the modem, monitor and notebook respectively

$$S_{Modem} = \begin{bmatrix} 0.47 & 0.17 & 0.13 & 0.43 & 0.20 & -0.04 \\ 0.17 & 0.24 & 0.14 & 0.18 & 0.36 & 0.06 \\ 0.13 & 0.14 & 0.97 & 0.17 & -0.29 & 0.26 \\ 0.43 & 0.18 & 0.17 & 1.17 & 0.26 & -0.38 \\ 0.20 & 0.36 & -0.29 & 0.26 & 1.62 & 0.12 \\ -0.04 & 0.06 & 0.26 & -0.38 & 0.12 & 0.85 \end{bmatrix};$$

$$S_{Monitor} = \begin{bmatrix} 1.45 & -0.03 & 0.60 & 1.53 & 1.00 & 0.99 \\ -0.03 & 0.19 & 0.16 & -0.16 & -0.04 & -0.10 \\ 0.60 & 0.16 & 1.74 & 1.02 & 1.11 & 1.07 \\ 1.53 & -0.16 & 1.02 & 2.18 & 1.46 & 1.31 \\ 1.00 & -0.04 & 1.11 & 1.46 & 1.21 & 1.02 \\ 0.99 & -0.10 & 1.07 & 1.31 & 1.02 & 1.37 \end{bmatrix};$$

$$S_{Notebook} = \begin{bmatrix} 0.11 & -0.08 & 0.38 & 0.07 & -0.12 & 0.15 \\ -0.08 & 0.60 & -0.11 & 0.69 & 0.42 & -0.11 \\ 0.38 & -0.11 & 3.24 & -0.43 & -0.14 & 0.40 \\ 0.07 & 0.69 & -0.43 & 2.54 & 0.61 & 0.24 \\ -0.12 & 0.42 & -0.14 & 0.61 & 0.61 & -0.30 \\ 0.15 & -0.11 & 0.40 & 0.24 & -0.30 & 0.66 \end{bmatrix}.$$