國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（一） **MATH 103A / GEAI 1215A: Linear Algebra I**

第一次期中考 October 2, 2023 **Midterm 1**

姓名 Name : solution

學號 Student ID # : _____

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page, **5 pages** of questions, score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. [5pt] Consider the three points

$$A = (2, 2, 2, 0, 0, 0),$$
$$B = (0, 0, 0, 2, 2, 2),$$
$$C = (2, 2, 2, 2, 2, 2).$$

Draw the triangle ABC on this paper as accurate as possible. Mark the length of the three sides and calculate the three angles.

$\overline{AB} = \|(2,2,2,-2,-2,-2)\| = \sqrt{4+4+4+4+4+4} = 2\sqrt{6}$.

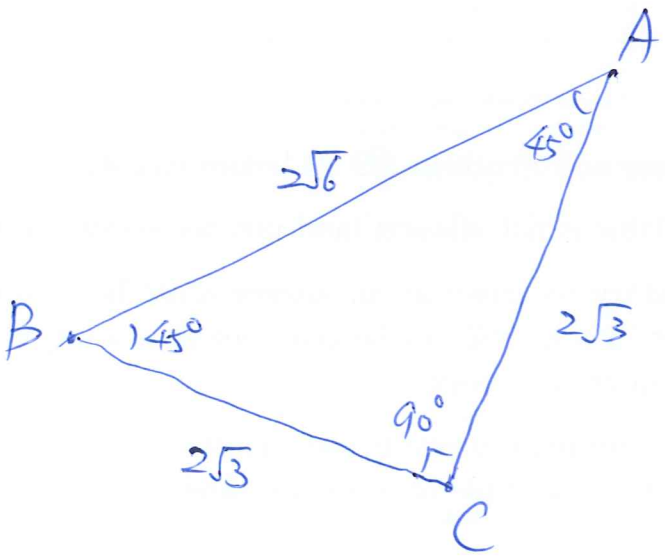
$\overline{AC} = \|(0,0,0,-2,-2,-2)\| = \sqrt{4+4+4} = 2\sqrt{3}$

$\overline{BC} = \|(-2,-2,-2,0,0,0)\| = \sqrt{4+4+4} = 2\sqrt{3}$.

Thus, ABC is a triangle with sides

$2\sqrt{3} : 2\sqrt{3} : 2\sqrt{6} = 1 : 1 : \sqrt{2}$,

so it's a ~~40° 4~~ 45°-45°-~~66°~~90° triangle.



2. Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 \\ 1 & 3 & 1 & 3 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 2 \\ 3 \\ -2 \\ -3 \end{bmatrix}, \quad \text{and } \mathbf{y} = \begin{bmatrix} 2 \\ 3 \\ 2 \\ 3 \end{bmatrix}.$$

(a) [1pt] Is $\mathbf{x}$ in $\ker(A)$?

Yes. $\vec{x} \in \ker(A)$ since $A\vec{x} = \vec{0}$.

(b) [1pt] Is $\mathbf{y}$ in $\ker(A)$?

No, $\vec{y} \notin \ker(A)$ since $A\vec{y} = \begin{bmatrix} 10 \\ 16 \\ 22 \end{bmatrix}$.

(c) [1pt] Is $\mathbf{x}$ in $\mathrm{Row}(A)$?

No, we may try to solve

$c_1 \times (1\ 1\ 1\ 1)$
$c_2 \times (1\ 2\ 1\ 2)$
$c_3 \times (1\ 3\ 1\ 3)$
$(2\ 3\ -2\ -3)$

and find no solution.

(d) [1pt] Is $\mathbf{y}$ in $\mathrm{Row}(A)$?

Yes, we may solve

$c_1 \times (1\ 1\ 1\ 1)$
$c_2 \times (1\ 2\ 1\ 2)$
$c_3 \times (1\ 3\ 1\ 3)$
$(2\ 3\ 2\ 3)$

and get $\vec{y} = 1 \cdot \text{row}\,1 + 1 \cdot \text{row}\,2$.

(e) [1pt] Describe the relation between $\ker(A)$ and $\mathrm{Row}(A)$.

They are orthogonal.

3. [5pt] Find all solutions of the following system of linear equations.

$$\begin{cases} x - 2y + 5u = 1 \\ 2x - 4y + z - 3w + 9u = 3 \\ -8x + 16y - 3z + 9w - 37u = -11 \end{cases}$$

See ver A.

4. [5pt] Mathematical essay: Write a few paragraphs to introduce the notion of $\text{span}(S)$.

   Your score will be based on the following criteria.

   - The definition is clear.
   - Some sentences are added to explain the definition.
   - Examples or pictures are included to help understanding.
   - The sentences are complete.

5. [extra 2pt] Let $\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{w}$ be vectors in $\mathbb{R}^n$ such that $\mathbf{w} = \mathbf{x} + \mathbf{y} + \mathbf{z}$. Show that $\mathbf{p} = 100\mathbf{x} + 200\mathbf{y} + 300\mathbf{z}$ is in $\text{span}(\{\mathbf{x}, \mathbf{y}, \mathbf{w}\})$.

See ver. A.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	