姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 12 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 5 \\ 29 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 2 \\ 3 \\ 19 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} -12 \\ -28 \\ -164 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 4 & -4 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 3-th column.

Therefore, $k = 3$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} 2 \\ 3 \\ 19 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 4.

ColBasis 1

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

4

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ -5 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 5 \\ -4 \\ -21 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -4 \\ 4 \\ 20 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} -19 \\ 16 \\ 83 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & -4 & -4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 3-th column.

Therefore, $k = 3$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} -4 \\ 4 \\ 20 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 0.

ColBasis 2

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

0

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 13 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 4 \\ 9 \\ 56 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -8 \\ -19 \\ -116 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} 3 \\ 6 \\ 39 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 4 & 3 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 3-th column.

Therefore, $k = 3$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} -8 \\ -19 \\ -116 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 7.

ColBasis 3

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

7

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -4 \\ 16 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -3 \\ 13 \\ -53 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -4 \\ 17 \\ -69 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} -10 \\ 42 \\ -170 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & -1 & -4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 3-th column.

Therefore, $k = 3$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} -4 \\ 17 \\ -69 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 4.

ColBasis 4

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

4

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} 5 \\ 16 \\ 7 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 2-th column.

Therefore, $k = 2$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 5.

ColBasis 5

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

5

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -5 \\ 10 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -4 \\ 20 \\ -40 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 2 \\ -10 \\ 20 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} -1 \\ 6 \\ -13 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \dots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \dots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & -4 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 2-th column.

Therefore, $k = 2$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} -4 \\ 20 \\ -40 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 6.

ColBasis 6

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

6

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -4 \\ 19 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -3 \\ 12 \\ -57 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -5 \\ 21 \\ -99 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} -24 \\ 101 \\ -476 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & -3 & 0 & 1 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 2-th column.

Therefore, $k = 2$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} -3 \\ 12 \\ -57 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 2.

ColBasis 7

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

2

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -4 \\ 5 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -5 \\ 21 \\ -27 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 15 \\ -63 \\ 81 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} 11 \\ -46 \\ 59 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 3-th column.

Therefore, $k = 3$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} 15 \\ -63 \\ 81 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 3.

ColBasis 8

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

3

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 20 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 4 \\ -12 \\ 80 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -3 \\ 10 \\ -65 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} 3 \\ -10 \\ 65 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 2-th column.

Therefore, $k = 2$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} 4 \\ -12 \\ 80 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 2.

ColBasis 9

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

2

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 4 \\ -13 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -5 \\ -20 \\ 65 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 4 \\ -13 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} 2 \\ 9 \\ -28 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & -5 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 2-th column.

Therefore, $k = 2$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} -5 \\ -20 \\ 65 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 0.

ColBasis 10

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

0