國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

**線性代數（一） MATH 103A / GEAI 1215A: Linear Algebra I**

**第一次期中考** October 3, 2022 **Midterm 1**

姓名 Name : ____________________

學號 Student ID # : ____________________

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| Lecturer: | Jephian Lin 林晉宏 |
| Contents: | cover page, |
| | **5 pages** of questions, |
| | score page at the end |
| To be answered: | on the test paper |
| Duration: | **110 minutes** |
| Total points: | **20 points** + 2 extra points |

**Do not open this packet until instructed to do so.**

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. Let

$$\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \text{ and } \mathbf{v} = \begin{bmatrix} 5 \\ -1 \\ -3 \\ 1 \end{bmatrix}.$$

(a) [1pt] Find the length of $\mathbf{u}$.

(b) [1pt] Let $\theta$ be the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$. Find $\cos\theta$.

(c) [1pt] Find a vector in $\text{span}(\{\mathbf{u}, \mathbf{v}\})$. Provide your reasons.

(d) [1pt] Find a vector **NOT** in $\text{span}(\{\mathbf{u}, \mathbf{v}\})$. Provide your reasons.

(e) [1pt] Find a nonzero vector that is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

2. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & -4 & -6 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}.$$

(a) [2pt] Draw the vector $\mathbf{b}$ and the subspace $\text{Col}(A)$ in $\mathbb{R}^2$. Mark at least one vector in $\text{Col}(A)$ in your drawing. Does $A\mathbf{x} = \mathbf{b}$ have a solution for $\mathbf{x}$?

(b) [1pt] Draw the subspace $\ker(A)$ and its normal vector in $\mathbb{R}^3$.

(c) [2pt] Find $\mathbf{h}_1$ and $\mathbf{h}_2$ such that $\ker(A) = \text{span}(\{\mathbf{h}_1, \mathbf{h}_2\})$.

3. Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 7 \\ 2 & 4 & 7 & 15 \\ -1 & -2 & -2 & -6 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} -7 \\ -17 \\ 4 \end{bmatrix}.$$

(a) [2pt] Find the reduced echelon form of the augmented matrix $\left[\begin{array}{c|c} A & \mathbf{b} \end{array}\right]$.

(b) [3pt] Find $\mathbf{p}$, $\mathbf{h}_1$, and $\mathbf{h}_2$ such that the set of general solutions of $A\mathbf{x} = \mathbf{b}$ is

$$\{\mathbf{p} + c_1\mathbf{h}_1 + c_2\mathbf{h}_2 : c_1, c_2 \in \mathbb{R}\}.$$

4. Suppose you are talking to people who have never learned linear algebra. Follow the guidelines below and try to explain the concept of a *subspace* as clear as possible.

   (a) [3pt] Define what is a subspace and use a few sentences to explain the definition.

   (b) [2pt] Provide an example of a subspace and an example of a set that is not a subspace. Provide your reasons.

5. [extra 2pt] Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 3 & 3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 3 \\ 5 \\ t \end{bmatrix}.$$

Find $t$ such that the equation $A\mathbf{x} = \mathbf{b}$ is consistent.

**[END]**

| Page | Points | Score |
|---|---|---|
| 1 | 5 | |
| 2 | 5 | |
| 3 | 5 | |
| 4 | 5 | |
| 5 | 2 | |
| Total | 20 (+2) | |