$\qquad$
$\qquad$

Let $G$ be the graph on 5 vertices as shown below．


Count the number of spanning trees of $G$ ．
Check code $=($ number of spanning trees $) \bmod 10$

## Solution．

The Laplacian matrix of $G$ is

$$
L=\left[\begin{array}{ccccc}
3 & -1 & 0 & -1 & -1 \\
-1 & 2 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 & -1 \\
-1 & 0 & 0 & 1 & 0 \\
-1 & -1 & -1 & 0 & 3
\end{array}\right] .
$$

Let $L^{\prime}$ be the matrix obtained from $L$ by removing the first row and the first column．Then the number of spanning trees is $\left|\operatorname{det}\left(L^{\prime}\right)\right|=3$ ．

Check code $=$（number of spanning trees） $\bmod 10=3$ ．

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Quiz 3

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MATH 207：Discrete Mathematics II

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Count the number of spanning trees of $G$ ．
Check code $=($ number of spanning trees $) \bmod 10$

## Solution．

The Laplacian matrix of $G$ is

$$
L=\left[\begin{array}{ccccc}
1 & 0 & -1 & 0 & 0 \\
0 & 3 & -1 & -1 & -1 \\
-1 & -1 & 2 & 0 & 0 \\
0 & -1 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 & 1
\end{array}\right] .
$$

Let $L^{\prime}$ be the matrix obtained from $L$ by removing the first row and the first column．Then the number of spanning trees is $\left|\operatorname{det}\left(L^{\prime}\right)\right|=1$ ．

Check code $=($ number of spanning trees $) \bmod 10=1$ ．

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Count the number of spanning trees of $G$ ．
Check code $=($ number of spanning trees $) \bmod 10$

## Solution．

The Laplacian matrix of $G$ is

$$
L=\left[\begin{array}{ccccc}
2 & -1 & 0 & -1 & 0 \\
-1 & 4 & -1 & -1 & -1 \\
0 & -1 & 1 & 0 & 0 \\
-1 & -1 & 0 & 3 & -1 \\
0 & -1 & 0 & -1 & 2
\end{array}\right] .
$$

Let $L^{\prime}$ be the matrix obtained from $L$ by removing the first row and the first column．Then the number of spanning trees is $\left|\operatorname{det}\left(L^{\prime}\right)\right|=8$ ．

Check code $=($ number of spanning trees $) \bmod 10=8$ ．

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Count the number of spanning trees of $G$ ．
Check code $=($ number of spanning trees $) \bmod 10$

## Solution．

The Laplacian matrix of $G$ is

$$
L=\left[\begin{array}{ccccc}
1 & -1 & 0 & 0 & 0 \\
-1 & 3 & -1 & -1 & 0 \\
0 & -1 & 3 & -1 & -1 \\
0 & -1 & -1 & 2 & 0 \\
0 & 0 & -1 & 0 & 1
\end{array}\right] .
$$

Let $L^{\prime}$ be the matrix obtained from $L$ by removing the first row and the first column．Then the number of spanning trees is $\left|\operatorname{det}\left(L^{\prime}\right)\right|=3$ ．

Check code $=($ number of spanning trees $) \bmod 10=3$ ．

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Count the number of spanning trees of $G$ ．
Check code $=($ number of spanning trees $) \bmod 10$

## Solution．

The Laplacian matrix of $G$ is

$$
L=\left[\begin{array}{ccccc}
2 & 0 & -1 & 0 & -1 \\
0 & 1 & 0 & 0 & -1 \\
-1 & 0 & 3 & -1 & -1 \\
0 & 0 & -1 & 1 & 0 \\
-1 & -1 & -1 & 0 & 3
\end{array}\right] .
$$

Let $L^{\prime}$ be the matrix obtained from $L$ by removing the first row and the first column．Then the number of spanning trees is $\left|\operatorname{det}\left(L^{\prime}\right)\right|=3$ ．

Check code $=$（number of spanning trees） $\bmod 10=3$ ．

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Let $G$ be the graph on 5 vertices as shown below．


Count the number of spanning trees of $G$ ．
Check code $=($ number of spanning trees $) \bmod 10$

## Solution．

The Laplacian matrix of $G$ is

$$
L=\left[\begin{array}{ccccc}
4 & -1 & -1 & -1 & -1 \\
-1 & 3 & -1 & -1 & 0 \\
-1 & -1 & 3 & -1 & 0 \\
-1 & -1 & -1 & 4 & -1 \\
-1 & 0 & 0 & -1 & 2
\end{array}\right] .
$$

Let $L^{\prime}$ be the matrix obtained from $L$ by removing the first row and the first column．Then the number of spanning trees is $\left|\operatorname{det}\left(L^{\prime}\right)\right|=40$ ．

Check code $=($ number of spanning trees $) \bmod 10=0$ ．

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Let $G$ be the graph on 5 vertices as shown below．


Count the number of spanning trees of $G$ ．
Check code $=($ number of spanning trees $) \bmod 10$

## Solution．

The Laplacian matrix of $G$ is

$$
L=\left[\begin{array}{ccccc}
1 & 0 & 0 & 0 & -1 \\
0 & 3 & -1 & -1 & -1 \\
0 & -1 & 3 & -1 & -1 \\
0 & -1 & -1 & 2 & 0 \\
-1 & -1 & -1 & 0 & 3
\end{array}\right] .
$$

Let $L^{\prime}$ be the matrix obtained from $L$ by removing the first row and the first column．Then the number of spanning trees is $\left|\operatorname{det}\left(L^{\prime}\right)\right|=8$ ．

Check code $=($ number of spanning trees $) \bmod 10=8$ ．

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Let $G$ be the graph on 5 vertices as shown below．


Count the number of spanning trees of $G$ ．
Check code $=($ number of spanning trees $) \bmod 10$

## Solution．

The Laplacian matrix of $G$ is

$$
L=\left[\begin{array}{ccccc}
1 & 0 & -1 & 0 & 0 \\
0 & 3 & -1 & -1 & -1 \\
-1 & -1 & 3 & 0 & -1 \\
0 & -1 & 0 & 1 & 0 \\
0 & -1 & -1 & 0 & 2
\end{array}\right] .
$$

Let $L^{\prime}$ be the matrix obtained from $L$ by removing the first row and the first column．Then the number of spanning trees is $\left|\operatorname{det}\left(L^{\prime}\right)\right|=3$ ．

Check code $=($ number of spanning trees $) \bmod 10=3$ ．

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Let $G$ be the graph on 5 vertices as shown below．


Count the number of spanning trees of $G$ ．
Check code $=($ number of spanning trees $) \bmod 10$

## Solution．

The Laplacian matrix of $G$ is

$$
L=\left[\begin{array}{ccccc}
2 & -1 & 0 & 0 & -1 \\
-1 & 1 & 0 & 0 & 0 \\
0 & 0 & 2 & -1 & -1 \\
0 & 0 & -1 & 1 & 0 \\
-1 & 0 & -1 & 0 & 2
\end{array}\right] .
$$

Let $L^{\prime}$ be the matrix obtained from $L$ by removing the first row and the first column．Then the number of spanning trees is $\left|\operatorname{det}\left(L^{\prime}\right)\right|=1$ ．

Check code $=($ number of spanning trees $) \bmod 10=1$ ．

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Let $G$ be the graph on 5 vertices as shown below．


Count the number of spanning trees of $G$ ．
Check code $=($ number of spanning trees $) \bmod 10$

## Solution．

The Laplacian matrix of $G$ is

$$
L=\left[\begin{array}{ccccc}
3 & -1 & -1 & -1 & 0 \\
-1 & 1 & 0 & 0 & 0 \\
-1 & 0 & 3 & -1 & -1 \\
-1 & 0 & -1 & 3 & -1 \\
0 & 0 & -1 & -1 & 2
\end{array}\right] .
$$

Let $L^{\prime}$ be the matrix obtained from $L$ by removing the first row and the first column．Then the number of spanning trees is $\left|\operatorname{det}\left(L^{\prime}\right)\right|=8$ ．

Check code $=($ number of spanning trees $) \bmod 10=8$ ．

