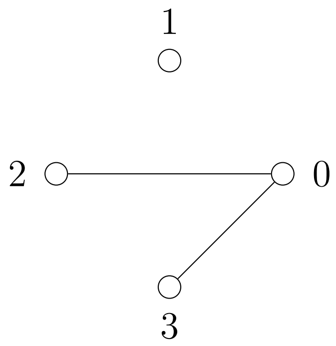
姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 207: Discrete Mathematics II

Let H be the graph on 4 vertices as shown below.



Let X be a random variable whose value is the number of induced subgraphs in the random graph model $G(n,p)$ that is isomorphic to H. Find a, b, c, d so that

$$\mathbb{E}(X) = a\binom{n}{b}p^c(1-p)^d.$$

Check code = $(a + b + c + d) \bmod 10$

Solution.

There are 2 automorphisms on H, so there are $4!/2 = 12$ ways to draw an unlabeled H on 4 labeled vertices.

There are $\binom{n}{4}$ ways to pick 4 vertices. On this set of vertices, the probability of getting an H is $12 \times p^2(1-p)^4$ since there are 2 edges and 4 nonedges in H.

Therefore,

$$\boxed{\mathbb{E}(X) = 12\binom{n}{4}p^2(1-p)^4}$$

with $a = 12$, $b = 4$, $c = 2$, and $d = 4$.

Check code = $(a + b + c + d) \bmod 10 = 2$.

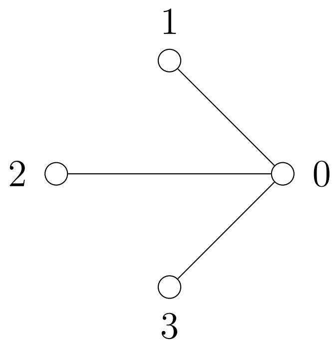
GnpSubG 1

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

2

Let H be the graph on 4 vertices as shown below.



Let X be a random variable whose value is the number of induced subgraphs in the random graph model $G(n, p)$ that is isomorphic to H. Find a, b, c, d so that

$$\mathbb{E}(X) = a\binom{n}{b}p^c(1-p)^d.$$

Check code = $(a + b + c + d) \bmod 10$

Solution.

There are 6 automorphisms on H, so there are $4!/6 = 4$ ways to draw an unlabeled H on 4 labeled vertices.

There are $\binom{n}{4}$ ways to pick 4 vertices. On this set of vertices, the probability of getting an H is $4 \times p^3(1-p)^3$ since there are 3 edges and 3 nonedges in H.

Therefore,

$$\boxed{\mathbb{E}(X) = 4\binom{n}{4}p^3(1-p)^3}$$

with $a = 4$, $b = 4$, $c = 3$, and $d = 3$.

Check code = $(a + b + c + d) \bmod 10 = 4$.

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

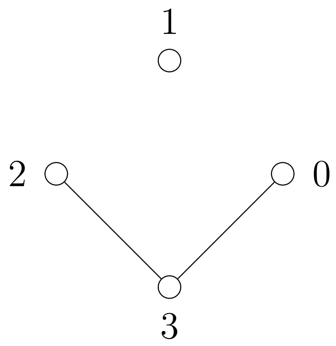
check code

4

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 207: Discrete Mathematics II

Let H be the graph on 4 vertices as shown below.



Let X be a random variable whose value is the number of induced subgraphs in the random graph model $G(n,p)$ that is isomorphic to H. Find a, b, c, d so that

$$\mathbb{E}(X) = a\binom{n}{b}p^c(1-p)^d.$$

Check code = $(a+b+c+d)$ mod 10

Solution.

There are 2 automorphisms on H, so there are $4!/2 = 12$ ways to draw an unlabeled H on 4 labeled vertices.

There are $\binom{n}{4}$ ways to pick 4 vertices. On this set of vertices, the probability of getting an H is $12 \times p^2(1-p)^4$ since there are 2 edges and 4 nonedges in H.

Therefore,

$$\boxed{\mathbb{E}(X) = 12\binom{n}{4}p^2(1-p)^4}$$

with $a = 12$, $b = 4$, $c = 2$, and $d = 4$.

Check code = $(a+b+c+d)$ mod 10 = 2.

GnpSubG 3

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

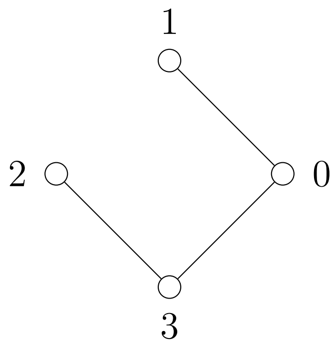
check code

2

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 207: Discrete Mathematics II

Let H be the graph on 4 vertices as shown below.



Let X be a random variable whose value is the number of induced subgraphs in the random graph model $G(n,p)$ that is isomorphic to H. Find a,b,c,d so that

$$\mathbb{E}(X) = a\binom{n}{b}p^c(1-p)^d.$$

Check code = $(a+b+c+d) \bmod 10$

Solution.

There are 2 automorphisms on H, so there are $4!/2 = 12$ ways to draw an unlabeled H on 4 labeled vertices.

There are $\binom{n}{4}$ ways to pick 4 vertices. On this set of vertices, the probability of getting an H is $12 \times p^3(1-p)^3$ since there are 3 edges and 3 nonedges in H.

Therefore,

$$\boxed{\mathbb{E}(X) = 12\binom{n}{4}p^3(1-p)^3}$$

with $a = 12$, $b = 4$, $c = 3$, and $d = 3$.

Check code = $(a+b+c+d) \bmod 10 = 2$.

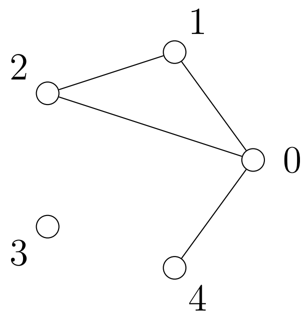
GnpSubG 4

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

2

Let H be the graph on 5 vertices as shown below.



Let X be a random variable whose value is the number of induced subgraphs in the random graph model $G(n,p)$ that is isomorphic to H. Find a, b, c, d so that

$$\mathbb{E}(X) = a\binom{n}{b}p^c(1-p)^d.$$

Check code $= (a+b+c+d) \bmod 10$

Solution.

There are 2 automorphisms on H, so there are $5!/2 = 60$ ways to draw an unlabeled H on 5 labeled vertices.

There are $\binom{n}{5}$ ways to pick 5 vertices. On this set of vertices, the probability of getting an H is $60 \times p^4(1-p)^6$ since there are 4 edges and 6 nonedges in H.

Therefore,

$$\boxed{\mathbb{E}(X) = 60\binom{n}{5}p^4(1-p)^6}$$

with $a = 60$, $b = 5$, $c = 4$, and $d = 6$.

Check code $= (a+b+c+d) \bmod 10 = 5$.

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

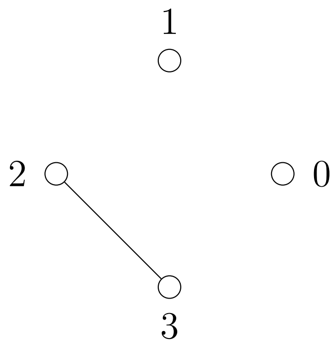
check code

5

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 207: Discrete Mathematics II

Let H be the graph on 4 vertices as shown below.



Let X be a random variable whose value is the number of induced subgraphs in the random graph model $G(n, p)$ that is isomorphic to H. Find a, b, c, d so that

$$\mathbb{E}(X) = a\binom{n}{b}p^c(1-p)^d.$$

Check code = $(a + b + c + d) \bmod 10$

Solution.

There are 4 automorphisms on H, so there are $4!/4 = 6$ ways to draw an unlabeled H on 4 labeled vertices.

There are $\binom{n}{4}$ ways to pick 4 vertices. On this set of vertices, the probability of getting an H is $6 \times p^1(1-p)^5$ since there are 1 edges and 5 nonedges in H.

Therefore,

$$\boxed{\mathbb{E}(X) = 6\binom{n}{4}p^1(1-p)^5}$$

with $a = 6$, $b = 4$, $c = 1$, and $d = 5$.

Check code = $(a + b + c + d) \bmod 10 = 6$.

GnpSubG 6

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

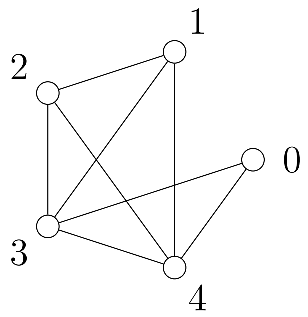
check code

6

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 207: Discrete Mathematics II

Let H be the graph on 5 vertices as shown below.



Let X be a random variable whose value is the number of induced subgraphs in the random graph model $G(n, p)$ that is isomorphic to H. Find a, b, c, d so that

$$\mathbb{E}(X) = a\binom{n}{b}p^c(1-p)^d.$$

Check code $= (a + b + c + d) \bmod 10$

Solution.

There are 4 automorphisms on H, so there are $5!/4 = 30$ ways to draw an unlabeled H on 5 labeled vertices.

There are $\binom{n}{5}$ ways to pick 5 vertices. On this set of vertices, the probability of getting an H is $30 \times p^8(1-p)^2$ since there are 8 edges and 2 nonedges in H.

Therefore,

$$\boxed{\mathbb{E}(X) = 30\binom{n}{5}p^8(1-p)^2}$$

with $a = 30$, $b = 5$, $c = 8$, and $d = 2$.

Check code $= (a + b + c + d) \bmod 10 = 5$.

GnpSubG 7

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

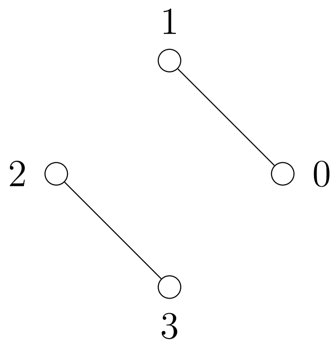
check code

5

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 207: Discrete Mathematics II

Let H be the graph on 4 vertices as shown below.



Let X be a random variable whose value is the number of induced subgraphs in the random graph model $G(n,p)$ that is isomorphic to H. Find a, b, c, d so that

$$\mathbb{E}(X) = a\binom{n}{b}p^c(1-p)^d.$$

Check code $= (a+b+c+d) \bmod 10$

Solution.

There are 8 automorphisms on H, so there are $4!/8 = 3$ ways to draw an unlabeled H on 4 labeled vertices.

There are $\binom{n}{4}$ ways to pick 4 vertices. On this set of vertices, the probability of getting an H is $3 \times p^2(1-p)^4$ since there are 2 edges and 4 nonedges in H.

Therefore,

$$\boxed{\mathbb{E}(X) = 3\binom{n}{4}p^2(1-p)^4}$$

with $a = 3$, $b = 4$, $c = 2$, and $d = 4$.

Check code $= (a+b+c+d) \bmod 10 = 3$.

GnpSubG 8

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

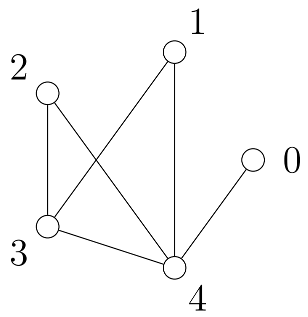
check code

3

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 207: Discrete Mathematics II

Let H be the graph on 5 vertices as shown below.



Let X be a random variable whose value is the number of induced subgraphs in the random graph model $G(n, p)$ that is isomorphic to H. Find a, b, c, d so that

$$\mathbb{E}(X) = a\binom{n}{b}p^c(1-p)^d.$$

Check code = $(a + b + c + d)$ mod 10

Solution.

There are 2 automorphisms on H, so there are $5!/2 = 60$ ways to draw an unlabeled H on 5 labeled vertices.

There are $\binom{n}{5}$ ways to pick 5 vertices. On this set of vertices, the probability of getting an H is $60 \times p^6(1-p)^4$ since there are 6 edges and 4 nonedges in H.

Therefore,

$$\boxed{\mathbb{E}(X) = 60\binom{n}{5}p^6(1-p)^4}$$

with $a = 60$, $b = 5$, $c = 6$, and $d = 4$.

Check code = $(a + b + c + d)$ mod 10 = 5.

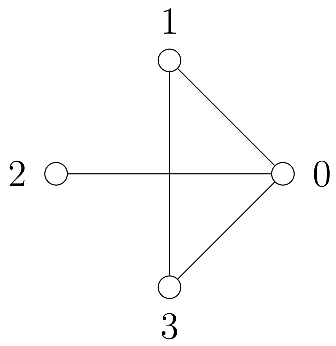
GnpSubG 9

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

5

Let H be the graph on 4 vertices as shown below.



Let X be a random variable whose value is the number of induced subgraphs in the random graph model $G(n,p)$ that is isomorphic to H. Find a,b,c,d so that

$$\mathbb{E}(X) = a\binom{n}{b}p^c(1-p)^d.$$

Check code $= (a+b+c+d) \bmod 10$

Solution.

There are 2 automorphisms on H, so there are $4!/2 = 12$ ways to draw an unlabeled H on 4 labeled vertices.

There are $\binom{n}{4}$ ways to pick 4 vertices. On this set of vertices, the probability of getting an H is $12 \times p^4(1-p)^2$ since there are 4 edges and 2 nonedges in H.

Therefore,

$$\boxed{\mathbb{E}(X) = 12\binom{n}{4}p^4(1-p)^2}$$

with $a = 12$, $b = 4$, $c = 4$, and $d = 2$.

Check code $= (a+b+c+d) \bmod 10 = 2.$

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

2