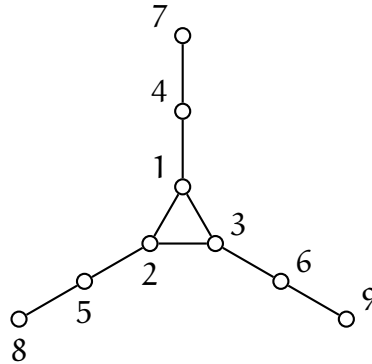# 2021F Math585 Midterm 2

5 questions, $20(+5)$ **total points**
Note: Use other papers to answer the problems. Remember to write down your **name** and your **student ID #**.

1. [5pt] Let G be the graph below and A its adjacency matrix. Find $\text{rank}(A)$ and $\det(A)$.

2. [5pt] Let G be a graph and A its adjacency matrix. Suppose the characteristic polynomial of A is
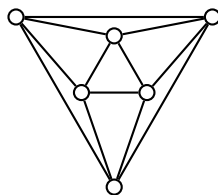$$(-x)^5 - 4(-x)^3 + 2(-x).$$
Find G.

3. [5pt] Let G be a graph and A its adjacency matrix. Suppose G is a graph containing an induced subgraph that is isomorphic to K_k. Prove that A has at least $k - 1$ eigenvalues that are at most -1.

Two more problems on the back.

4. [5pt] Let G be the graphs below and A its adjacency matrix.

Find the eigenvalues of A.

5. [extra 5pt] Let
$$A = \begin{bmatrix} O & B & O \\ O & O & C \\ D & O & O \end{bmatrix},$$

where O is the 3×3 zero matrix and B, C, D are some fixed 3×3 matrices. Let $\omega = e^{i\frac{2\pi}{3}}$. Show that if λ is an eigenvalue of A, then $\omega\lambda$ is also an eigenvalue of A.