# 2021F Math585 Midterm1

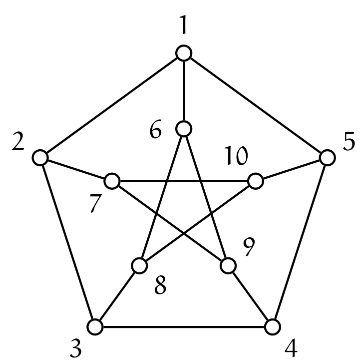
**5 questions, 20 total points**
**Note:** Use other papers to answer the problems. Remember to write down your **name** and your **student ID #**.

1. [5pt] Let

$$A = \begin{bmatrix} 0 & x & 0 & 0 & 0 & 0 & 0 & 1 \\ x & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Find $x$ such that the $1,5$-entry of $A^4$ is $0$.

2. [5pt] Let $G$ be the Petersen graph and $A$ its adjacency matrix as shown below.




$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$G$ $A$

Let $S_k$ be the sum of all $k \times k$ principal minors of $A$. Find $S_5$ and explain your reasons.

3. [5pt] Let $J_n$ and $I_n$ be the $n \times n$ all-ones matrix and the identity matrix of order $n$, respectively. Let $D_n = J_n - I_n$. Find the inertia of $D_n$ and explain your reasons.

**One more problem on the back.**

4. [5pt] Let $A$ be a $7 \times 7$ real symmetric matrix. Let $\{\mathbf{v}_1, \ldots, \mathbf{v}_7\}$ be an orthonormal eigenbasis of $A$ such that $A\mathbf{v}_i = \lambda_i \mathbf{v}_i$ for $i = 1, \ldots, 7$ and $\lambda_1 \leqslant \cdots \leqslant \lambda_7$. Consider the space $W = \operatorname{span}\{v_2, v_4, v_6\}$. Show that

$$\lambda_2 = \min_{\substack{\mathbf{x} \in W \\ \mathbf{x} \neq \mathbf{0}}} \frac{\mathbf{x}^\top A \mathbf{x}}{\mathbf{x}^\top \mathbf{x}}.$$