姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103 / GEAI 1215: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 7 \\ 11 \\ -23 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} 3 \\ 6 \\ -15 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 4 & 3 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 3-th column.

Therefore, $k = 3$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} 7 \\ 11 \\ -23 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 5.

ColBasis 1

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

5

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103 / GEAI 1215: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 4 \\ 11 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 5 \\ 20 \\ 55 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -5 \\ -20 \\ -55 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} -5 \\ -19 \\ -53 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & 5 & -5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 2-th column.

Therefore, $k = 2$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} 5 \\ 20 \\ 55 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 0.

ColBasis 2

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

0

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103 / GEAI 1215: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -5 \\ 19 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -5 \\ 26 \\ -98 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -5 \\ 26 \\ -98 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} -6 \\ 31 \\ -117 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 3-th column.

Therefore, $k = 3$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} -5 \\ 26 \\ -98 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 3.

ColBasis 3

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

3

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103 / GEAI 1215: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 5 \\ 18 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -1 \\ -4 \\ -15 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 0 \\ -4 \\ -12 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} 4 \\ 15 \\ 57 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & -4 & -1 \\ 0 & 1 & -4 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 3-th column.

Therefore, $k = 3$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} 0 \\ -4 \\ -12 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 4.

ColBasis 4

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

4

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103 / GEAI 1215: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 5 \\ -5 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 3 \\ 15 \\ -15 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 3 \\ 16 \\ -15 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} 18 \\ 95 \\ -90 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 2-th column.

Therefore, $k = 2$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} 3 \\ 15 \\ -15 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 3.

ColBasis 5

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

3

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103 / GEAI 1215: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 4 \\ -20 \\ 8 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} -3 \\ 16 \\ -6 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 2-th column.

Therefore, $k = 2$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} 4 \\ -20 \\ 8 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 2.

ColBasis 6

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

2

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103 / GEAI 1215: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -5 \\ 25 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -1 \\ 6 \\ -30 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ -10 \\ 50 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} 2 \\ -10 \\ 50 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \dots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \dots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & -4 & 2 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 3-th column.

Therefore, $k = 3$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} 1 \\ -10 \\ 50 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 1.

ColBasis 7

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

1

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103 / GEAI 1215: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -3 \\ -14 \\ -19 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -1 \\ -3 \\ -3 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} 11 \\ 50 \\ 67 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 5 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 3-th column.

Therefore, $k = 3$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} -1 \\ -3 \\ -3 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 3.

ColBasis 8

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

3

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103 / GEAI 1215: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ -4 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -5 \\ 1 \\ -25 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} -24 \\ 4 \\ -116 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & -1 & 0 & -4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 2-th column.

Therefore, $k = 2$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} -1 \\ 0 \\ -4 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 5.

ColBasis 9

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

5

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 2 MATH 103 / GEAI 1215: Linear Algebra I

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 4 \\ -14 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -1 \\ -4 \\ 14 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 3 \\ 13 \\ -46 \end{bmatrix}, \text{and } \mathbf{v}_4 = \begin{bmatrix} -8 \\ -35 \\ 124 \end{bmatrix}.$$

Find the vector $\mathbf{v}_k$ with the smallest k such that $\{\mathbf{v}_1, \dots, \mathbf{v}_{k-1}\}$ is linearly independent but $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$ is linearly dependent.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10

Solution.

Let $\mathbf{A}$ be the matrix whose columns are $\{\mathbf{v}_1, \dots, \mathbf{v}_4\}$. The reduced echelon form of $\mathbf{A}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the first free variable occurs on the 2-th column.

Therefore, $k = 2$ and $\boxed{\mathbf{v}_k = \begin{bmatrix} -1 \\ -4 \\ 14 \end{bmatrix}}$.

Check code = (sum of all entries of $\mathbf{v}_k$) mod 10 = 9.

ColBasis 10

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

9