國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（一） **MATH 103 / GEAI 1215: Linear Algebra I**

第一次期中考 November 1, 2021 **Midterm 1**

姓名 Name : _____

學號 Student ID # : _____

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page, **5 pages** of questions, score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- 可用中文或英文作答

1. Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \text{ and } \mathbf{p} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

(a) [1pt] Find a vector in $\mathrm{Row}(A)$ that is nowhere zero （每一項都不是零）.

(b) [1pt] Find a vector in $\mathrm{Col}(A)$ that is nowhere zero.

(c) [1pt] Find a vector in $\ker(A)$ that is nowhere zero.

(d) [1pt] Find a vector in $\mathbf{p} + \ker(A)$ that is nowhere zero.

(e) [1pt] Let

$$B = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}.$$

Find a 3×3 matrix E such that $EA = B$.

2. Let

$$A = \begin{bmatrix} 1 & -2 & 0 & 3 \\ 2 & -4 & 1 & 10 \\ 3 & -6 & 2 & 17 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} -4 \\ -13 \\ -22 \end{bmatrix}.$$

(a) [2pt] Find the reduced row echelon form of the augmented matrix $\left[\begin{array}{c|c} A & \mathbf{b} \end{array}\right]$.

(b) [3pt] Find $\mathbf{p}$, $\mathbf{h}_1$, $\mathbf{h}_2$ such that

$$\{\mathbf{x} \in \mathbb{R}^4 : A\mathbf{x} = \mathbf{b}\} = \mathbf{p} + \text{span}(\{\mathbf{h}_1, \mathbf{h}_2\}).$$

3. Let

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & -1 & -2 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

(a) [3pt] Find $\mathbf{w}$ and $\mathbf{h}$ such that $\mathbf{b} = \mathbf{w} + \mathbf{h}$ with $\mathbf{w} \in \text{Col}(A)$ and $\mathbf{h} \in \text{Col}(A)^{\perp}$.

(b) [2pt] Let θ be the angle between $\mathbf{b}$ and $\mathbf{w}$. Find $\cos\theta$.

4. [5pt] 數學作文：請寫一篇短文來向沒修過線性代數的朋友介紹什麼是子空間（subspace）。

   請以盡量白話的敘述、或是比喻來介紹什麼是子空間？爲什麼要考慮這樣的概念？並給一些能幫助他人理解的例子（正面的、反面的）；有必要的話可以加上一些圖來輔助說明。格式沒有限制，篇輻大約半面到一面。

   (If Chinese is not your native language, you may use English or the language that you prefer.)

5. [extra 2pt] Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Find a 5×5 matrix E such that $EAE^\top = I$.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	