國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（二） **MATH 104 / GEAI 1209: Linear Algebra II**

第二次期中考 May 4, 2020 **Midterm 2**

姓名 Name : ____________________

學號 Student ID # : ____________________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page, **7 pages** of questions, score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**30 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- 可用中文或英文作答

1. Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \text{ and } V = \operatorname{span}(\{\mathbf{v}_1, \mathbf{v}_2\}).$$

(a) [2pt] Find two vectors in $V^\perp$ such that they are independent.

(b) [1pt] Find a vector that is not in $V^\perp$.

2. Let

$$A = \begin{bmatrix} i & j & k & \ell \\ m & n & o & p \\ q & r & s & t \\ u & v & w & x \end{bmatrix}.$$

By the permutation expansion, $\det(A) = insx + \cdots$ is the sum of 4! terms.

(a) [1pt] Find a term of $\det(A)$ with positive sign (other than $insx$).

(b) [1pt] Find a term of $\det(A)$ with negative sign.

3. Let

$$V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + 3y + 2z = 0 \right\}.$$

(a) [2pt] Find a basis of V.

(b) [3pt] Let $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Find the projection of $\mathbf{u}$ onto the plane V.

4. [5pt] Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \text{ and } \mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

Apply the Gram–Schmidt algorithm to the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ and obtain an orthogonal basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ for $V = \operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

5. [5pt] Let

$$A = \begin{bmatrix} x & 3 & 0 & 0 \\ 3 & 0 & -1 & -1 \\ 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 \end{bmatrix},$$

where x is a real number. Find $x \in \mathbb{R}$ such that $\det(A) = 0$.

6. [5pt] Let

$$A = \begin{bmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & d & d^2 & d^3 \end{bmatrix}.$$

Show that $\det(A) = (b-a)(c-a)(d-a)(c-b)(d-b)(d-c)$. Make sure to justify every step of your argument.

7. [5pt] Let J_n be the $n \times n$ all-ones matrix. Let I_n be the $n \times n$ identity matrix. Find $\det(J_n - I_n)$ as a formula of n. Make sure to justify every step of your argument.

8. [extra 2pt] Let

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}.$$

Find $\det(A)$.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	2	
Total	30 (+2)	