## 2020F Math589 Midterm2

5 questions, 25 total points
Note: Use other papers to answer the problems. Remember to write down your name and your student ID \#.

1. [5pt] Consider the following four drawings and determine whether they are topologically isomorphic to each other. (E.g., (a)~(b), (c)~(d), but (a) $\sim$ (c).)

2. [5pt] It is known that every graph $\mathrm{H} \in \mathrm{I} K_{5}$ is also in $T K_{5}$ or in $T K_{3,3}$. For the following $\mathrm{H}_{1}, \mathrm{H}_{2} \in \mathrm{IK}_{5}$, show that they contains $\mathrm{K}_{5}$ or $\mathrm{K}_{3,3}$ as a topological minor.

3. [5pt] Find a graph $G$ and an edge $e \in E(G)$ such that $G$ is not planar but $G / e$ (contraction of G on $e$ ) is planar.
[Two more problems on the back.]
4. [5pt] Let $G$ be the graph below and $\mathcal{C}(G)$ its cycle space. Answer the following questions:
(a) How many elements are in $\mathcal{C}(G)$ ?
(b) What is the dimension of $\mathcal{C}(G)$ ?
(c) Find a basis of $\mathcal{C}(G)$.

5. [5pt] Consider the matrix below over $\mathbb{Z}_{2}$, the field of two elements.

$$
A=\left[\begin{array}{llllllll}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1
\end{array}\right]
$$

Find a basis of the row space of $A$.

