## 2020F Math589 Midterm1

## 5 questions, 25 total points

**Note:** Use other papers to answer the problems. Remember to write down your **name** and your **student ID #**.

- 1. [5pt] Draw a  $K_5$  on a torus.
- 2. [5pt] Find a  $K_{3,3}$  or a  $K_5$  as a minor of the graph below.



3. [5pt] Let  $\alpha(G)$  be the independence number of G, that is, the largest k such that there are k vertices in G that are not adjacent to each other. (For example,  $\alpha(K_{3,3}) = 3$ .) Define

 $[\alpha](G) = \max\{\alpha(H) : H \text{ is a minor of } G\}.$ 

Find  $\lceil \alpha \rceil(G)$  for each G.

- 4. [5pt] Prove that  $K_5$  and  $K_{3,3}$  are not planar using Euler's formula V E + F = 2.
- 5. [5pt] Prove that if X is a graph with  $\Delta(X) \leq 3$ , then any IX contains an TX as an subgraph.