## 2020F Math589 Midterm1

5 questions, 25 total points
Note: Use other papers to answer the problems. Remember to write down your name and your student ID \#.

1. [5pt] Draw a $\mathrm{K}_{5}$ on a torus.
2. [5pt] Find a $K_{3,3}$ or a $K_{5}$ as a minor of the graph below.

3. [5pt] Let $\alpha(G)$ be the independence number of $G$, that is, the largest $k$ such that there are $k$ vertices in $G$ that are not adjacent to each other. (For example, $\alpha\left(\mathrm{K}_{3,3}\right)=3$.) Define

$$
\lceil\alpha\rceil(\mathrm{G})=\max \{\alpha(\mathrm{H}): \mathrm{H} \text { is a minor of } \mathrm{G}\} .
$$

Find $\lceil\alpha\rceil(G)$ for each $G$.
4. [5pt] Prove that $K_{5}$ and $K_{3,3}$ are not planar using Euler's formula $V-E+F=2$.
5. [5pt] Prove that if $X$ is a graph with $\Delta(X) \leqslant 3$, then any IX contains an TX as an subgraph.

