

## Math589 Homework 4

1. [1pt] Use induction and Lemma 4.1.3 to prove any plane forest has exactly one face.  
**Solution.**

2. [1pt] Use the Jordan curve theorem and Lemma 4.2.1 to show that if a plane graph has only one face, then it is a forest.

**Solution.**

Questions to ponder:

1. Which plane graphs have exactly two faces?
2. What happens if a plane graph has two faces with the same frontier? What can we say about the graph?
3. Is the boundary of a face always a cycle? Give some examples.
4. If a graph is 2-connected, is the boundary of a face always a cycle?
5. Practice your  $\text{\TeX}$ nique at <https://texnique.xyz/>.
6. Let  $G$  and  $H$  be two graphs. Google how to use SageMath to find a subgraph of  $G$  that is  $IH$ , or return None. You may use SageCell to try your code.