Math589 Homework 15

1. [1pt] Recall that if $f(x_1, \ldots, x_n) = (f_1, \ldots, f_m)$, then

$$\frac{df}{d(x_1, \ldots, x_n)} = \begin{bmatrix} \frac{df_1}{dx_1} & \frac{df_1}{dx_2} & \cdots & \frac{df_1}{dx_n} \\ \vdots & \vdots & & \vdots \\ \frac{df_m}{dx_1} & \frac{df_m}{dx_2} & \cdots & \frac{df_m}{dx_n} \end{bmatrix}.$$

   Let $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{y} \in \mathbb{R}^m$ be two vectors and A an $m \times n$ matrix. Define $f(\mathbf{x}) = A\mathbf{x} + \mathbf{y}$. Find $\frac{df}{d\mathbf{x}}$.

   Solution.

2. [1pt] Let A be the adjacency matrix of $K_{1,4}$. Determine whether A has the strong Arnold property or not. If yes, verify it; if no, find a matrix X such that

$$A \circ X = I \circ X = AX = O.$$

Solution.

Questions to ponder:

1. Pick a symmetric matrix and check if it has the strong Arnold property or not.
2. Let $f(\mathbf{x}) = A\mathbf{x} + \mathbf{y}$. Find $\frac{df}{d\mathbf{y}}$.
3. Show that the zero forcing number Z is not minor-monotone.
4. Show that every matrix of $K_{1,3}$ has the strong Arnold property.
5. Show that every matrix of C_4 has the strong Arnold property.
6. Practice your TEXnique at `https://texnique.xyz/`.