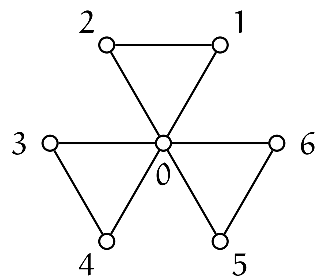
Math589 Homework 12

1. [1pt] Let G be the graph below. Recall that a fort on G is a subset $S \subseteq V(G)$ of vertices such that for any $x \in V(G) \setminus S$, the number of neighbors of x in S is either zero or at least two. Find all forts on G.



Solution.

2. [1pt] Let G be a graph. Show that B is a zero forcing set of G if and only if $B \cap S \neq \emptyset$ for all forts S of G.

   Solution.

Questions to ponder:

1. Show that $\delta(G) \leqslant Z(G)$ for all graph G.
2. Show that $|V(H)| - Z(H) \leqslant |V(G)| - Z(G)$ whenever H is an induced subgraph of G.
3. Show that for graphs without isolated vertices, $Z(G) \leqslant |V(G)| - \alpha(G)$, where $\alpha(G)$ is the independence number.
4. Let G_n be the graph obtained from C_n, the cycle on n vertices, by adding a leaf to each of the vertices on C_n. Find $Z(G_n)$.
5. Let G_1 and G_2 be two graphs. Pick a vertex on G_1 and a vertex on G_2, then label both of them as v. Let G be the graph obtained from $G_1 \cup G_2$ by identifying the two vertices labeled as v.
   (a) $Z(G) \leqslant Z(G_1) + Z(G_2) + 1$
   (b) $Z(G) \leqslant Z(G_1 - v) + Z(G_2 - v) + 1$
   (c) Find G_1, G_2, v such that both of the inequalities are not tight.
6. Practice your TEXnique at `https://texnique.xyz/`.