## Math589 Homework 1

1. [1pt] Find three graphs $G_{1}, G_{2}$, and $G_{3}$ such that

- $G_{1}$ is a minor but not a topological minor of $G_{3}$, and
- $\mathrm{G}_{2}$ is a topological minor of $\mathrm{G}_{3}$.

Use TikZ to illustrate your graphs and explain your reasons.

## Solution.

2. [1pt] Let $\omega(\mathrm{G})$ be the clique number, the largest size of a clique in G. Define $\lfloor\omega\rfloor(G)=\min \{\omega(H): H$ is $I G\}$. Find $\lfloor\omega\rfloor(G)$ for each $G$.

## Solution.

Questions to ponder:

1. Prove that if $\Delta(G) \leqslant 3$, then any IG contains an TG.
2. Describe an algorithm to test whether $H$ is in IG. When $G$ is small, e.g., $K_{1}, K_{2}, K_{3}$, $P_{2}, P_{3}$, is it easier?
3. Practice your $\mathrm{T}_{\mathrm{E}}$ Xnique at https://texnique. $x y z /$.
