## Math589 Homework 1

1. [1pt] Find three graphs  $G_1$ ,  $G_2$ , and  $G_3$  such that

- $G_1$  is a minor but not a topological minor of  $G_3$ , and
- $G_2$  is a topological minor of  $G_3$ .

Use *TikZ* to illustrate your graphs and explain your reasons.

Solution.

2. [1pt] Let  $\omega(G)$  be the clique number, the largest size of a clique in G. Define  $\lfloor \omega \rfloor(G) = \min\{\omega(H) : H \text{ is } IG\}$ . Find  $\lfloor \omega \rfloor(G)$  for each G.

Solution.

Questions to ponder:

- 1. Prove that if  $\Delta(G) \leq 3$ , then any IG contains an TG.
- 2. Describe an algorithm to test whether H is in IG. When G is small, e.g.,  $K_1$ ,  $K_2$ ,  $K_3$ ,  $P_2$ ,  $P_3$ , is it easier?
- 3. Practice your T\_EXnique at https://texnique.xyz/.