國立中山大學   NATIONAL SUN YAT-SEN UNIVERSITY

離散數學（一）   **MATH 203: Discrete Mathematics I**

第二次期中考   November 24, 2020   **Midterm 2**

姓名 Name ： _solution_

學號 Student ID # ： _____

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	5 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.

- Using the calculator is not allowed (and not necessary) for this exam.

- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.

- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

- 可用中文或英文作答

1. Let G be the graph below. Answer the following questions and **provide your reasons**. [Hint: This graph is composed of the vertices and the edges of an icosahedron (正二十面體).]
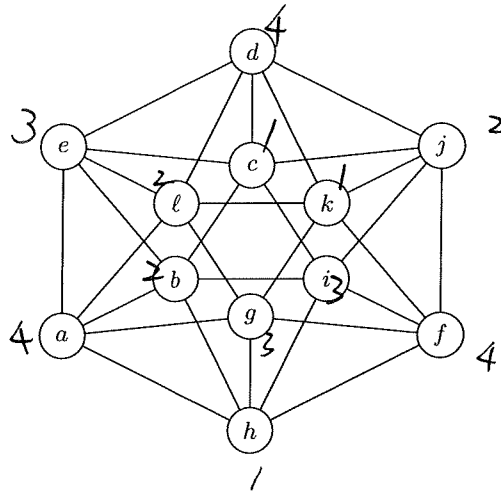
(a) [1pt] Is G a bipartite graph?

No, G contains a triangle.
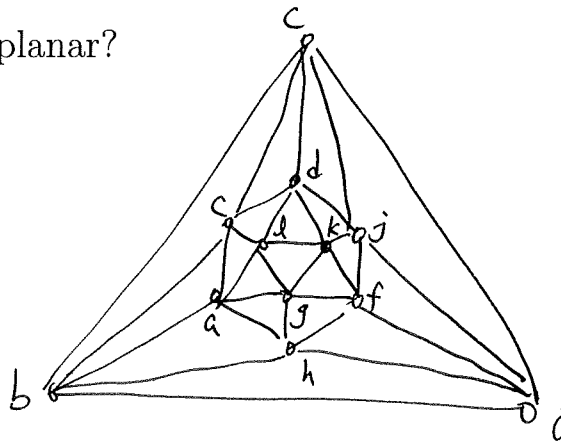
(b) [1pt] Is there an Eulerian circuit on G?

No, G contains vertices of odd degree.

(c) [1pt] Is there a Hamiltonian cycle on G?

Yes, e.g. $d - e - a - h - g - \ell - k - j - f - i - d - c - d$.
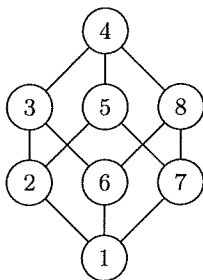
(d) [1pt] Is G planar?

Yes,

(e) [1pt] Is G 4-colorable?

Yes, see above.

2. Let (X, R) be the poset with the Hasse diagram below. Answer the following questions and **provide your reasons**.

(a) [1pt] Find a linear extension of (X, R).

~~$1 \leq 2 \leq 3 \leq 4$~~ $1 \leq 2 \leq 6 \leq 7 \leq 3 \leq 5 \leq 8 \leq 4$.

(b) [1pt] Find a total order on $\{1, \ldots, 8\}$ that is not a linear extension of (X, R).
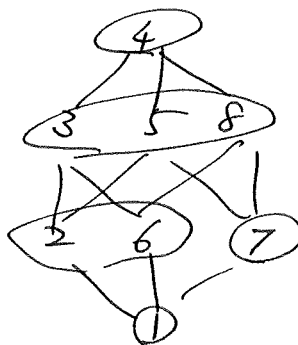
$1 \leq 2 \leq \ldots \leq 8$.

since $6 \leq 3$ in R.

(c) [1pt] Is there a chain cover of (X, R) of size 2?

No, $\{3, 5, 8\}$ is an antichain of size 3.

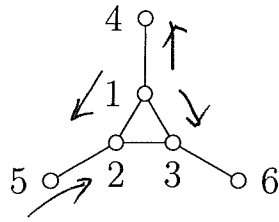(d) [1pt] Is there an antichain cover of (X, R) of size 5?

Yes, e.g.

(e) [1pt] Is (X, R) an interval poset?

No, $\begin{matrix} 3 \\ | \\ 2 \end{matrix} \quad \begin{matrix} 8 \\ | \\ 7 \end{matrix}$ is a $2 + 2$.

3. [5pt] Determine whether the graph below is a comparability graph or not and **provide your reasons.**

$$4 \circ \uparrow$$
$$\swarrow 1 \circ \searrow$$
$$5 \circ \nearrow \; 2 \quad 3 \; \circ 6$$

Assume $1 \to 4$.

$\vdash 4 \;\Rightarrow\; \begin{array}{l} 1 \to 2 \\ 1 \to 3 \end{array}$

$\begin{array}{l} 1 \to 2 \;\Rightarrow\; 5 \to 2 \;\Rightarrow\; 3 \to 2 \\ 1 \to 3 \;\Rightarrow\; 6 \to 3 \;\Rightarrow\; 2 \to 3 \end{array} \Bigg] \to \!\!\!\! \times .$

So the graph is not a comparability graph.

4. [5pt] Let (X, R) be a poset. Recall that

$$D(x) = \{y \in X : y \preceq x \text{ in } R\}$$

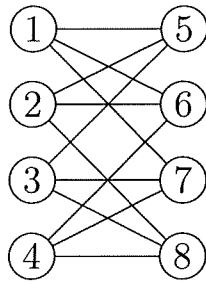for any $x \in X$ and $\mathbf{2} + \mathbf{2}$ is the poset whose Hasse diagram is as below.
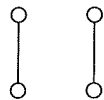
Show that the following statements are equivalent:

(a) (X, R) contains $\mathbf{2} + \mathbf{2}$ as a subposet.

(b) There are two elements $x_1, x_2 \in X$ such that $D(x_1) \setminus D(x_2) \neq \emptyset$ and $D(x_2) \setminus D(x_1) \neq \emptyset$.

See   ver. A.

5. [extra 2pt] Let G be the graph below. Find an order of the vertices (e.g., $8, 7, \ldots, 1$) such that the greedy coloring algorithm using this order needs 4 colors.

See ver. A.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	