國立中山大學   NATIONAL SUN YAT-SEN UNIVERSITY

離散數學（一）   **MATH 203: Discrete Mathematics I**

第一次期中考   October 13, 2020   **Midterm 1**

姓名 Name : _____

學號 Student ID # : _____

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	5 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.

- Using the calculator is not allowed (and not necessary) for this exam.

- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.

- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

- 可用中文或英文作答

1. [5pt] Recall that $H_k^n = \binom{n+k-1}{k}$ counts the number of integer solutions of

$$x_1 + \cdots + x_n = k, \quad x_i \geq 0$$

for all $i = 1, \ldots, n$. **Use double counting** to prove that

$$H_k^{n+1} = H_k^n + H_{k-1}^n + \cdots + H_0^n.$$

2. [5pt] **Use mathematical induction** to prove that

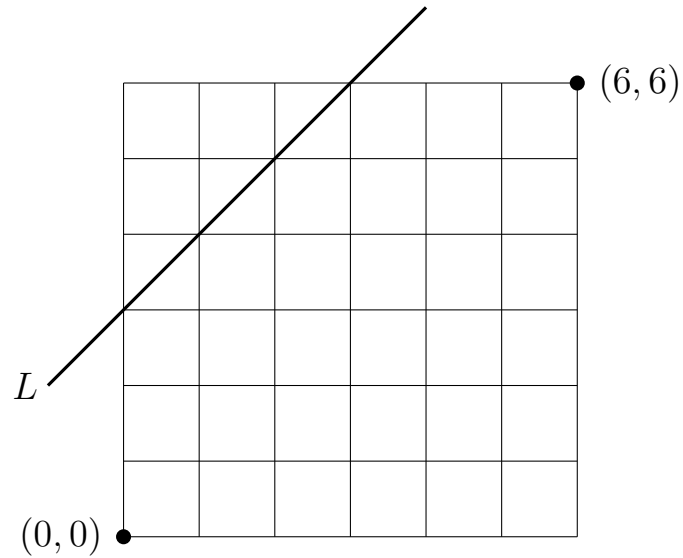$$n^3 + 2n \text{ is divisible by 3 for all integer } n \geq 1.$$

3. [5pt] Prove that any set $S \subseteq \{1, \ldots, 140\}$ with $|S| = 71$ contains two numbers a and b such that a is divisible by b.

4. [5pt] Let $m = 2020$ and $n = 109$. Find integers a and b such that

$$am + bn = 1.$$

5. [extra 2pt] Consider two possible moves $\rightarrow$: $(1,0)$ and $\uparrow$: $(0,1)$. Count the number of ways to go from $(0,0)$ to $(6,6)$ such that

- each step is either $\rightarrow$ or $\uparrow$, and
- it **touches** the line $L : y = x + 3$. (有碰到就算)

$(6,6)$

L

$(0,0)$

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	