Math589 Midterm1

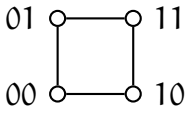
6 questions, 24 total points
Note: Use other papers to answer the problems. Remember to write down your **name** and your **student ID #**.

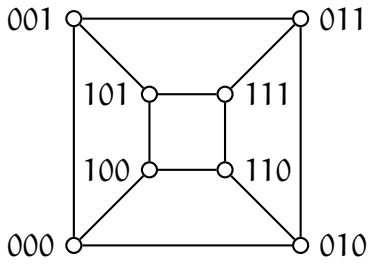
1. [4pt] Show that every simple graph G must have two vertices whose degrees are the same.

2. [4pt] Suppose G is a connected simple graph on n vertices and m edges. Show that $m \geqslant n - 1$.

3. [4pt] Find all connected graphs on 5 vertices. How many of them?

4. [4pt] The *Hamming distance* between two $0,1$-strings is the number of different digits. For example, the Hamming distance between 010101 and 111000 is 3. The *hypercube* H_d of dimension d has vertices as all $0,1$-strings of length d, and two vertices are adjacent if the Hamming distance of the strings is 1. The graphs below illustrate H_2 and H_3. Find a partition $X \dot\cup Y = V(H_d)$ so that every edge of H_d is in between X and Y.





5. [4pt] Let C_n be the cycle graph on n vertices and L_n the Laplacian matrix of C_n. Recall that $L_n(1,1)$ is the matrix obtained from L_n by removing the first row and the first column. Compute $|\det L_n(1,1)|$.

[One more problem on the back.]

6. Let G be the graph below. Find the chromatic number $k = \chi(G)$ and give a proper k-coloring of G.