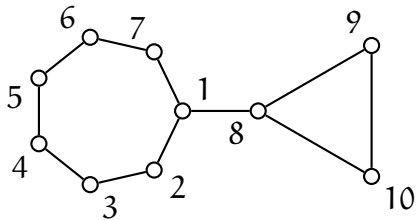# Math589 Homework 8

Note: To submit the k-th homework, simply put your files in the folder `HWk` on CoCalc, and it will be collected on the due day.

1. Let G be the graph drawn below. Find a balanced partition

$$V(G) = X_1 \mathbin{\dot\cup} X_2 \text{ with } |X_1| = |X_2|$$

   that minimizes the number of edges between X_1 and X_2.

   Solution. For example, take

$$X_1 = \{3, 4, 5, 6, 7\} \text{ and } X_2 = \{1, 2, 8, 9, 10\}.$$

2. Let G be the same graph as in Problem 1. Let $\mathbf{v}$ be the eigenvector corresponding to the second (smallest) eigenvalue. Find

$$\operatorname{supp}_+(\mathbf{v}) := \{i \in V(G) : (\mathbf{v})_i > 0\},$$
$$\operatorname{supp}_-(\mathbf{v}) := \{i \in V(G) : (\mathbf{v})_i < 0\},$$
$$\operatorname{supp}_0(\mathbf{v}) := \{i \in V(G) : (\mathbf{v})_i = 0\}.$$

You may use a computer if necessary.

Solution. By Sage, the eigenvector is

$$\begin{bmatrix} 1 \\ -2.200261711964298? \\ -4.877989230088837? \\ -6.397256299602498? \\ -6.397256299602498? \\ -4.877989230088837? \\ -2.200261711964298? \\ 7.163036132758463? \\ 9.39398917527641? \\ 9.39398917527641? \end{bmatrix}$$

or its multiple. Therefore,

$$\operatorname{supp}_+(\mathbf{v}) = \{1, 8, 9, 10\},$$
$$\operatorname{supp}_-(\mathbf{v}) = \{2, 3, 4, 5, 6, 7\},$$
$$\operatorname{supp}_0(\mathbf{v}) = \emptyset.$$