## Math589 Homework 8

Note: To submit the k-th homework, simply put your files in the folder HWk on CoCalc, and it will be collected on the due day.

1. Let G be the graph drawn below. Find a balanced partition

$$
V(G)=X_{1} \cup X_{2} \text { with }\left|X_{1}\right|=\left|X_{2}\right|
$$

that minimizes the number of edges between $X_{1}$ and $X_{2}$.

2. Let G be the same graph as in Problem 1. Let $\mathbf{v}$ be the eigenvector corresponding to the second (smallest) eigenvalue. Find

$$
\begin{aligned}
\operatorname{supp}_{+}(\mathbf{v}) & :=\left\{i \in \mathrm{~V}(\mathrm{G}):(\mathbf{v})_{i}>0\right\}, \\
\operatorname{supp}_{-}(\mathbf{v}) & :=\left\{i \in \mathrm{~V}(\mathrm{G}):(\mathbf{v})_{i}<0\right\}, \\
\operatorname{supp}_{0}(\mathbf{v}) & :=\left\{i \in \mathrm{~V}(\mathrm{G}):(\mathbf{v})_{i}=0\right\} .
\end{aligned}
$$

You may use a computer if necessary.

