## Math589 Homework 2

Note: To submit the k-th homework, simply put your files in the folder HWk on CoCalc, and it will be collected on the due day.

1. Suppose $G$ is a simple graph on $n$ vertices and $m$ edges. Show that if $m \leqslant n-2$ then $G$ is not connected.

Solution. Start from $n$ isolated vertices. (That is, a graph on $n$ vertices without any edge.) When an edge is added, it at most combines two components into one component. This operation decreases the number of components by at most one. If $m \leqslant n-2$ edges were added, then the number of components is at least $n-m \geqslant 2$, so $G$ is not connected.
2. Find two distinct graphs on 4 vertices with 3 edges.

Solution. Each of the following two graphs has 4 vertices and 3 edges.

$$
0-0-0-0
$$



