Math589 Homework 10

Note: To submit the k-th homework, simply put your files in the folder `HWk` on CoCalc, and it will be collected on the due day.

1. Recall the following two versions of the Borsuk–Ulam Theorem.

(BU1b) For every antipodal mapping $f : S^n \to \mathbb{R}^n$, there is a point $\mathbf{x} \in S^n$ such that $f(\mathbf{x}) = \mathbf{0}$.

(BU2a) There is no antipodal mapping $f : S^n \to S^{n-1}$.

Show that they are equivalent.

Solution. First we show (BU1b) $\Longrightarrow$ (BU2a). Suppose $f : S^n \to S^{n-1}$ is an antipodal mapping. Then f is also an antipodal mapping from S^n to $\mathbb{R}^n$, but there is no point $\mathbf{x} \in S^n$ such that $f(\mathbf{x}) = \mathbf{0}$ since $f(\mathbf{x}) \in S^{n-1}$, which is a contradiction.

Next we show (BU2a) $\Longrightarrow$ (BU1b). Suppose $f : S^n \to \mathbb{R}^n$ is an antipodal mapping such that there is no point $\mathbf{x} \in S^n$ with $f(\mathbf{x}) = \mathbf{0}$. Then $g(x) := f(x)/|f(x)|$ is well-defined and is an antipodal mapping from S^n to S^{n-1}, which is a contradiction to (BU2a).

2. Let T be a triangulation of B^2, as shown below, that is antipodally symmetric on the boundary. Label the vertices $V(T)$ by $\{\pm 1, \pm 2\}$ such that it is antipodal on the boundary, then indicate all complementary edges by red lines.

Solution.

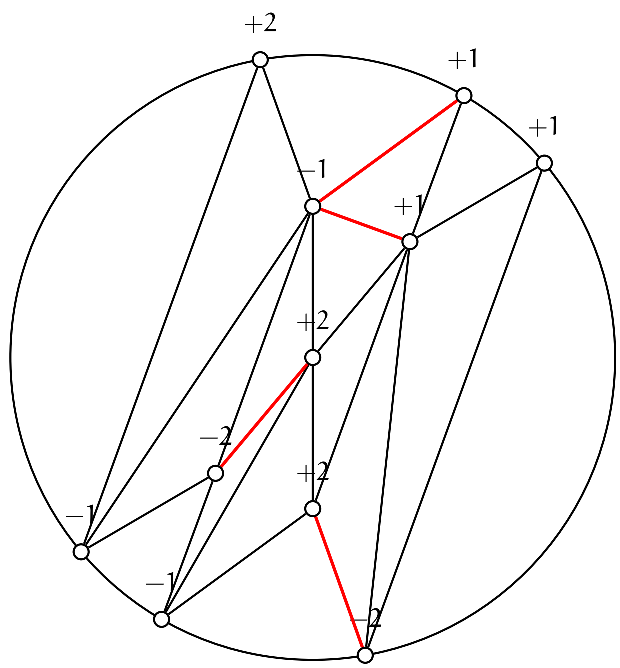

Figure 1: A triangulation T of B^2