Sample Questions 2

1. Let $\mathbf{A}$ be a matrix, $\mathbf{u}$, $\mathbf{v}$ two vectors, and r a real number. Show that $\mathbf{A}(\mathbf{u}+\mathbf{v}) = \mathbf{A}\mathbf{u} + \mathbf{A}\mathbf{v}$ and $\mathbf{A}(r\mathbf{v}) = r(\mathbf{A}\mathbf{v})$.

2. Let
$$\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$
Let
$$A = \{c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 : c_1, c_2, c_3 \in \mathbb{R}\}$$
and
$$B = \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} : x + y + z + w = 0 \right\}.$$
Show that $A = B$ by proving $\mathbf{v} \in A \implies \mathbf{v} \in B$ and $\mathbf{v} \in B \implies \mathbf{v} \in A$.)

3. Let $\mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ be a zero vector in $\mathbb{R}^n$. Consider it as an $n \times 1$ matrix. Show that applying any row operation on $\mathbf{0}$ will lead to $\mathbf{0}$. [Therefore, if $(\mathbf{A}|\mathbf{b})$ becomes $(\mathbf{R}|\mathbf{r})$ after some row operations, then $(\mathbf{A}|\mathbf{0})$ will be $(\mathbf{R}|\mathbf{0})$ after the same row operations.]

4. Find the general solution of the following linear system.
$$\begin{cases} 3x + 6y = 18 \\ x + 2y = 6 \end{cases}$$

5. Find the general solution of the following linear system.
$$\begin{cases} \phantom{w+{}} x + 2y - z = 3 \\ w + 2x + y \phantom{{}-z} = 4 \\ w + x - y + z = 1 \end{cases}$$

6. Find the general solution of the following linear system.
$$\begin{cases} u + w + x + y + z = 1 \\ 2u + 2w + 2x + 2y + 2z = 2 \end{cases}$$

7. For each of the following matrices, is it singular or nonsingular? Give your reason.

   (a) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \\ 12 & 13 & 14 & 15 \end{bmatrix}$