Sample Questions 3

1. Find the reduced echelon form of the augmented matrix and find the general solution of the following linear system.

$$\begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & -1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 5 \end{bmatrix}$$

2. Find the reduced echelon form of the augmented matrix and find the general solution of the following linear system.

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$$

3. Find the reduced echelon form of the augmented matrix and find the general solution of the following linear system.

$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 2 & -1 & 1 & 1 \\ 3 & 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

4. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 11 & 14 \\ -8 & -10 \end{bmatrix}$, and $C = \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$. You may verify that $CA = B$. Write $\begin{bmatrix} 11 & 14 \end{bmatrix}$ as a linear combination of rows of A.

5. The matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ has the reduced echelon form $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Write the row vector $\begin{bmatrix} 0 & 1 \end{bmatrix}$ as a linear combination of rows of A.

6. The matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 11 & 2 \\ 8 & 17 & 2 \end{bmatrix}$ has the reduced echelon form $R = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$. Write the row vector $\begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$ as a linear combination of rows of A.

7. Suppose A, B, and C are three matrices such that $AC = B$. Show that each column of B is a linear combination of columns of A.