Sample Questions 2

1. Let $\mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ be a zero vector in $\mathbb{R}^n$. Consider it as an $n \times 1$ matrix. Show that applying any row operation on $\mathbf{0}$ will lead to $\mathbf{0}$. [Therefore, if $(\mathbf{A}|\mathbf{b})$ becomes $(\mathbf{R}|\mathbf{r})$ after some row operations, then $(\mathbf{A}|\mathbf{0})$ will be $(\mathbf{R}|\mathbf{0})$ after the same row operations.]

2. Find the general solution of the following linear system.
$$\begin{cases} 3x + 6y = 18 \\ x + 2y = 6 \end{cases}$$

3. Find the general solution of the following linear system.
$$\begin{cases} \phantom{w+{}} x + 2y - z = 3 \\ w + 2x + y \phantom{{}-z} = 4 \\ w + x - y + z = 1 \end{cases}$$

4. Find the general solution of the following linear system.
$$\begin{cases} u + w + x + y + z = 1 \\ 2u + 2w + 2x + 2y + 2z = 2 \end{cases}$$

5. For each of the following matrices, is it singular or nonsingular? Give your reason.

   (a) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \\ 12 & 13 & 14 & 15 \end{bmatrix}$

6. Let
$$\mathbf{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \text{ and } X = \left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \end{bmatrix} \right\}.$$
Can $\mathbf{v}$ be written as a linear combination of vectors in X?

7. Let
$$\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \text{ and } X = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 2 \end{bmatrix} \right\}.$$
Can $\mathbf{v}$ be written as a linear combination of vectors in X?