Sample Questions 10

1. Let V be a subspace of $\mathbb{R}^n$. Show that $V^\perp$ is also a subspace.

2. Suppose W is the orthogonal complement of a subspace V. Show that $W^\perp = V$. That is,
$$(V^\perp)^\perp = V.$$

3. Let V be a subspace of $\mathbb{R}^n$. Show that
$$V^\perp \cap V = \{\mathbf{0}\}.$$

4. Let
$$V = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \\ 2 \\ 0 \end{bmatrix} \right\}.$$
Find a basis of $V^\perp$.

[For questions regarding the orthogonal projection, Sage can help you on the computation. But make sure you know how to do it by hand! Click here on the pdf for the code.]

5. Let
$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \text{ and } \mathbf{y} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}.$$
Find $\cos\theta$, where θ is the angle between $\mathbf{x}$ and $\mathbf{y}$, and then use the formula
$$|\mathbf{x}| \cdot \cos\theta \cdot \frac{\mathbf{y}}{|\mathbf{y}|}$$
to find the orthogonal projection of $\mathbf{x}$ onto $\mathbf{y}$. Moreover, what is the orthogonal projection of $\mathbf{x}$ onto the hyperplane
$$\left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} : x - y + z - w = 0 \right\}.$$

6. Note that
$$V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + 2y + 3z = 0 \right\} = \operatorname{span}\left\{ \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} \right\}.$$
Find the orthogonal projection of $\mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$ onto V. [You may solve it using what you learned in class or in high school.]

7. Let V be the vector space defined in Problem 4 and $\mathbf{x}$ the vector defined in Problem 5. Find the orthogonal projection of $\mathbf{x}$ onto V.