國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（一） **MATH 103 / GEAI 1215: Linear Algebra I**

第一次期中考 October 8, 2018 **Midterm 1**

姓名 Name ： _____

學號 Student ID # ： _____

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	6 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**30 points** + 2 extra points

SAMPLE

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

1. [1pt] Suppose $S = \{\vec{z_1}, \vec{z_2}, \ldots, \vec{z_n}\}$ is a set of n vectors over $\mathbb{R}$. Write down the definition of that "$\vec{v}$ is a *linear combination* of vectors in S."

2. [1pt] Suppose $\vec{p} = (p_1, \ldots, p_n)$ and $\vec{q} = (q_1, \ldots, q_n)$ are two vectors in $\mathbb{R}^n$. Write down the definition of the *inner product* of $\vec{p}$ and $\vec{q}$.

3. [2pt] Give a linear system in echelon form with two free variables, and indicate the free variables.

4. [2pt] Give two 3×3 matrices such that one is singular while the other is nonsingular.

5. Let
$$\vec{\boldsymbol{u}} = (0, 1, -1, -1, -1) \text{ and}$$
$$\vec{\boldsymbol{v}} = \left(\sqrt{3}, \frac{2+3\sqrt{3}}{2}, \frac{-2+\sqrt{3}}{2}, \frac{-2+\sqrt{3}}{2}, \frac{-2+\sqrt{3}}{2}\right).$$

   (a) [1pt] Find the length $\|\vec{\boldsymbol{u}}\|$.

   (b) [1pt] Find the length $\|\vec{\boldsymbol{v}}\|$.

   (c) [2pt] Find the angle between $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$.

6. [2pt] Let $\vec{\boldsymbol{p}} = (1, 2, 3, 4, 5)$. Find a vector $\vec{\boldsymbol{q}}$ that is parallel to $\vec{\boldsymbol{p}}$ and a vector $\vec{\boldsymbol{r}}$ that is orthogonal to $\vec{\boldsymbol{p}}$. [Note: "parallel" means the angle is 0 or π; "orthognal" means the angle is $\frac{\pi}{2}$.]

7. [6pt] Find the general solution of the following linear system.

$$\begin{cases} w - x + y - z = -2 \\ 2w - 2x + y + z = 5 \\ 3w - 3x + 2y = 3 \end{cases}$$

That is, find $\vec{\boldsymbol{p}}$ and $\overrightarrow{\boldsymbol{\beta_1}}, \ldots, \overrightarrow{\boldsymbol{\beta_k}}$ such that

$$\{\vec{\boldsymbol{p}} + c_1\overrightarrow{\boldsymbol{\beta_1}} + \cdots + c_k\overrightarrow{\boldsymbol{\beta_k}} : c_1, \ldots, c_k \in \mathbb{R}\}$$

is the set of all solutions.

8. [6pt] Find the reduced echelon form of

$$\begin{bmatrix} 1 & 1 & 5 & 2 \\ 1 & 2 & 7 & -3 \\ -2 & -2 & -10 & -3 \end{bmatrix}.$$

9. [6pt] Let

$$A = \begin{bmatrix} 1 & -2 & 3 & -4 \\ -1 & 2 & -3 & 5 \\ 2 & -4 & 6 & -8 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

It is known that $\boldsymbol{R}$ can be obtained from $\boldsymbol{A}$ by performing some row operations. Find a matrix $\boldsymbol{C}$ such that $\boldsymbol{CA} = \boldsymbol{R}$.

10. [extra 2pt] ???

[END]

Page	Points	Score
1	6	
2	6	
3	6	
4	6	
5	6	
6	2	
Total	30 (+2)	