translated by Alessandro Rosa - *released* on May 15th, 2001

Glossary

This is a simple list of the most recurrent concepts, terms and theorems appearing in this article; they belong to topology, discrete dynamical systems and complex analysis.
Every term is described as exhaustively as possible.
I hope this may help the reader in reading and learning this article.

Analytic
A complex function is analytic when it always preserves the same amplification factor and the same angle of rotation for every *source point* and in any direction.
So any analytic function is conformal.

Analytic continuation
It's a special technique used in complex analysis to expand the analyticity of a function. For example, given a function G(*z*) converging in the unit circle and diverging outside it, then the *analytic continuation* consists empirically in finding a new function H(*z*) which is still analytic (conformal) in the domain of G(*z*) exactly in the same way as G(*z*) but beyond too. In this way, H(*z*) is said to be the *analytic continuation* of G(*z*).

Annulus
A closed region between two concentric circles.

Attractor
Any invariant set to which all nearby orbits converge. Attractors therefore describe the long-term behavior of a dynamical system.

Automorphism
A one-to-one, conformal mapping of a set to itself.

Backward asymptotic
Read *Forward asymptotic*.

Basin of attraction
It's the set of all initial points, whose orbits approach to an *attractor*. This concept may be applied both to a single point and to a *periodic orbit*; in this last case there are *n* basins of attraction for *n periodic points*, respectively. All the *n* basins of the periodic points generate the *basin of attraction of the cycle*. There is the *total basin* including all points approaching to an attractor. The *immediate basin* is the set connected component belonging to the *total basin*.

Boundary
Given a set A, it's the set coming from the subtraction of the *interior* set from the *closure* set : $Fr\ A = \overline{A} - Int\ A$.

Branch
A *branch point* (or just *branch*) is a point *z* such that given a initial value *z* = *a* and *f*(*z*) = *b*, then *f* (*z*) fails to come back to *b* as *z* travels along any loop encircling *a* once.
Branch points could be: *algebraic branch point* of an order (N - 1) if *f*(*z*) comes back to the original value *b* after N revolutions around *a*, *simple point* if it is an *algebraic branch point of order* 1, finally it is a *logarithmic branch point* if *f*(*z*) never comes back to *b*.

Closed set
A set whose complement is an open set.

Closure
Given a set A, it's the smallest set including A.
Usually is denoted $\overline{A}$.

Connectedness.
A space enjoys the property of connectedness and it is said to be *connected* if it cannot be represented as the union $X_1 \cup X_2$ of two non-empty disjoint closed subsets. Trivially a *connected* set consists of points, so that any arbitrary pair can be linked by an unbroken curve lying entirely in the same set.
Connectedness is an invariant of continuous mappings.
There are some types of connectedness: *simply connected*, *multiply connected*, *path-wise connected*.

Conformal map
If the angle between the every image curves is the same as the angle between the every original curve through a point *p*, then the map has "preserved" the angle at *p*. Such map is denoted as *conformal* and it enjoys *conformality*.

Contour
Synonym of *boundary*.

Critical point
A point *p* where the conformality of *f* breaks down. So a point, where the first order derivative vanishes, is denoted *critical*. A critical point is said to be non-degenerate if $f''(z) \neq 0$. So the critical point is degenerate if $f''(z) = 0$. Critical points cannot occur for diffeomorphisms.

Critical value
The image of a critical point.

Cyclic orbit
Read *Periodic orbit*.

Degenerate point
Read *Critical point*.

Diffeomorphism
If a given function *f* is an *homeomorphism* and *f* is differentiable too. There different orders of diffeomorphism, according to the same orders of differentiability of *f*.

Disconnected set
A set that does not enjoy *connectedness*.

ε - chain
Let F be a diffeomorphism. A point x is chain recurrent for F, if, for any $\varepsilon > 0$, there are points $x = x_0, x_1, x_2, ..., x_k = x$ and positive integers $n_1, ..., n_k$ such that

$$| F_{ni} (x_i - 1) - x_i | < \varepsilon$$

for each i.
The sequence of points $x_0, ..., x_n$ is called *ε - chain* or a presudo-orbit. Intuitively, an *ε - chain* is almost an orbit in the sense that we allow small jumps or errors at iterations n_1, $n_1 + n_2$, etc. Note that the *ε - chains* always begin and end at x.

Essential singularity
When the singular point is neither a *removable singularity* nor a *pole*. For example in the function $\sin\left(\frac{1}{z}\right)$, which in the interval $0 < z < 1$ has wilder and wilder behaviour as it approaches to zero from the right.

Eventually periodic
When the point z is not periodic but it compares in an orbit that will get in a periodic cycle .

Function
Read *Map*.

Fixed point
A point whose image by f is itself : $f(z) = z$. A fixed point is a periodic point with period 1.

Flower
Read *Petal*.

Forward asymptotic
A point z is forward asymptotic to p if $\lim_{i\to\infty} f^{in}(z) = p$. If p is non-periodic, the forward asymptotic points may be defined by requiring $| f^i(z) - f^i(p) | \to 0$ as $i \to\infty$.
Also, if f is invertible, we may consider *backward asymptotic* points by letting $i \to -\infty$ (the opposite direction of *forward asymptotic* points).

Holomorphic
Synonym of *Analytic*.

Homeomorphism
If a given function f is *one-to-one*, onto and continuous and the f^{-1} is also continuous.
An homeomorphism cannot produce periodic orbits.

Homographic mapping
Synonym of "bilinear", "linear", "linear-fractional".

Hyperbolicity
This property may be enjoyed by a periodic point p, so that the following relation holds $| (f^n)'(p) | \neq 1$.
The number $(f^n)'(p)$ is called multiplier of the p. A point enjoying *hyperbolicity* is called *hyperbolic*.

Image
The set of points retrieved by $w = f(z)$ when z assumes a range of values in the domain (read definition of *preimage*).

Initial point
The first point of an orbit; usually is indexed at zero : z_0.

Interior
Given a set A, it's the largest open set included in A.

Invariant
An object which is mapped onto itself. It's *strictly invariant* if the object is mapped exactly to itself. It's *completely* invariant if it's *strictly invariant* for both direct and inverse mappings.

Invertible maps
One-to-one maps, where a point has one and only one preimage. Read *One-to-one*.

Isolated singularity
A singular point (or singularity) where its sufficiently small neighbourhood includes no other singularities. An isolated singularity of an analytic function is either a *pole* or an *essential singularity*.

Layer set
A set where all its points have the same number of preimages. Sometimes it's called as *multiplicity zone*.

Map, mapping
A continuous function from a space, called *domain*, to itself.

Multiplicity zone
Read *Layer set*.

Multiply connected
A set is multiply connected if it is connected where there are two distinct paths linking two points so that one path cannot be deformed into the other. Trivially a *simply connected set* includes at least one "hole".

Neighbourhood
Given a set X and a point $x \in X$, then U is a neighbourhood of x, if $U \subset X$ and $x \in U$.

Neutrality
This property is enjoyed by a periodic point p, so that the following relation holds $| (f^n)'(p) | = 1$.
The number $(f^n)'(p)$ is called multiplier of the p. A point enjoying *neutrality* is called *neutral* or *indifferent*. Neutral points may be distinguished in: *rational indifferent* (if it is a rational number) or *irrational indifferent* (if it is irrational).

Non-invertible
Many-to-one maps, where a point has many preimages.

Normal family
Let $\{F_n\}$ be a family of complex analytic functions defined on an open set U. Then $\{F_n\}$ is normal on U if every sequence of the F_n's has a subsequence which either converges uniformly on compact subsets of U or it converges uniformly to ∞ on U.

One-to-one
Given a map $f : U \to W$, then every element of U always corresponds to one and only one element of W.

Orbit
An infinite of finite set of points generated by repeating applications of a mapping.

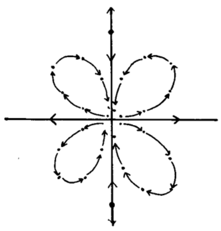
Period
The integer number of periodic points. A period is said *prime* for the least positive integer *n*.

Periodic cycle
Read *periodic orbit*.

Periodic orbit
A type of orbit consisting of a finite set of points, so that the map permutes the points of the orbit cyclically.
Each of these points comes back to its original position after exactly *n* iteration of the map.
So a periodic orbit consists of periodic points.
Synonym of *periodic cycle*.

Periodic point
A point reoccurring in the orbit $f^n(z) = z$, with period n .

Petal
It occurs often in cardioid shaped regions including neutral points. A simply connected region C is an *attracting petal* for an indifferent (neutral) fixed point z_0 so that if z_0 is included in the boundary of C and for each $z \in C$, then $P^n(z) \to z_0$.



A *repelling petal is defined analogously*.
Each type of petal may lie aside the other type so that the dynamics of the orbits in these two regions describe shapes that look like petals of a *flower* (look at the picture on the right).

Picard's Theorem
(Little theorem): Let *f* be an entire function and suppose that the range of *f* omits two distinct complex values α and β. Then *f* must be identically costant.
(Great theorem): Let U be a region in the plane and a point $P \in U$; suppose that *f* is holomorphic on U\{P} and has an essential singularity at P. If $\varepsilon > 0$, then the restriction of *f* to $U \cap D(P,\varepsilon)\setminus\{P\}$ assumes all complex values except possibly one.

Pole
Given any function F(z), the pole is the preimage of ∞.

Preimage
The point retrieved by $z = f^{-1}(w)$ when *w* assumes a range of values in the image set (read definition of *image*).

Trapping
An interval which is mapped into itself.

Relation
Read *Map*.

Removable singularity
Given a function $f(z)$ and a singular point *p* then *p* is *removable* when $\lim_{z\to p} f(z)$ exists.

Singular point
General term to denote the point where the function behaves atypically; for example a *pole*.

Simply connected
A set is simply connected if it is connected so that a path, linking an arbitrary pair of points of the same set, keeps on belonging to the set under the action of a continuous deformation.
Trivially a *simply connected set* includes no "holes".

Source point
Any point belonging to the domain set.

Stable set
The stable set of *p* , denoted by $W^s(p)$, consists of all points forward asymptotic to *p*.

Unstable set
The unstable set of *p* , denoted by $W^u(p)$, consists of all points backward asymptotic to *p*.