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The Extended Tanh-Method For Finding Traveling Wave Solutions Of Nonlinear Evolution Equations*

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Abstract

In this article, we find traveling wave solutions of the coupled (2+1)-dimensional Nizhnik-Novikov-Veselov and the (1+1)-dimensional Jaulent-Miodek (JM) equations. Based on the extended tanh method, an efficient method is proposed to obtain the exact solutions to the coupled nonlinear evolution equations. The extended tanh method presents a wider applicability for handling nonlinear wave equations.

1 Introduction

The investigation of the traveling wave solutions of nonlinear partial differential equations plays an important role in the study of nonlinear physical phenomena. Nonlinear wave phenomena appears in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics and geochemistry. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. In recent years, new exact solutions may help us find new phenomena. A variety of powerful methods, such as the inverse scattering method [1, 13], bilinear transformation [7], tanh-sech method [10, 11], extended tanh method [5, 10], homogeneous balance method [5] and Jacobi elliptic function method [15] were used to develop nonlinear dispersive and dissipative problems. The pioneer work of Malfiet in [10, 11] introduced the powerful tanh method for reliable treatment of the nonlinear wave equations. The useful tanh method is widely used by many authors such as [17–20] and the references therein. Later, the extended tanh method, developed by Wazwaz [21, 22], is a direct and effective algebraic method for handling nonlinear equations. Various extensions of the method were developed as well. The next interest is in the determination of the exact traveling wave solutions for the coupled (2+1)-dimensional Nizhnik-Novikov-Veselov and the (1+1)-dimensional Jaulent-Miodek (JM) equations. Searching for the exact solutions of nonlinear problems has attracted a considerable amount of research work where computer symbolic systems facilitate the computational work. We implement

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the proposed method for the (2+1)-dimensional Nizhnik-Novikov-Veselov equations [16]

$$\begin{gathered} u_t + ku_{xxx} + ru_{yyy} + su_x + qu_y = 3k(uv)_x + 3r(uw)_y, \\ u_x = v_y, u_y = w_x, \end{gathered} \tag{1}$$

and the (1+1)-dimensional Jaulent-Miodek (JM) equations

$$\begin{gathered} u_t + u_{xxx} + \tfrac{3}{2}vv_{xxx} + \tfrac{9}{2}v_xv_{xx} - 6uu_x - 6uvv_x - \tfrac{3}{2}u_xv^2 = 0, \\ v_t + v_{xxx} - 6u_xv - 6uv_x - \tfrac{15}{2}v_xv^2 = 0, \end{gathered} \tag{2}$$

where k, r, s and q are arbitrary constants. In the past years, many people studied the Nizhnik-Novikov-Veselov equations. For instance, Pempinelli et al. [2] solved NNV equations via the inverse scattering transformation, Zhang et al. [14] and Zhang et al. [23] obtained the Jacobi elliptic function solution of the NNV equations by the sinh-cosh method. Lou [9] analyzed the coherent structures of the NNV equation by separation of variables approach. The coupled system of equations (2) associates with the JM spectral problem [8], the relation between this system and Euler-Darboux equation was found by Matsuno [12]. In recent years, much work associated with the JM spectral problems has been done [24, 25]. Fan [4] has investigated the exact solution of (2) using the unified algebraic method. Our first interest in the present work is in implementing the extended tanh method to stress its power in handling nonlinear equations so that one can apply it to models of various types of nonlinearity such as (1) and (2).

2 The Extended Tanh Method

Wazwaz has summarized the use of the extended tanh method. A PDE

$$P(u, u_t, u_x, u_{xx}, ...) = 0, \tag{3}$$

can be converted to the following ODE

$$Q(U, U', U'', U''', ...) = 0, \tag{4}$$

by means of a wave variable $\xi = x - \beta t$ so that $u(x,t) = U(\xi)$ and using the following change of variables (in the derivatives)

$$\frac{\partial}{\partial t} = -\beta\frac{d}{d\xi},\ \frac{\partial}{\partial x} = \frac{d}{d\xi},\ \frac{\partial^2}{\partial x^2} = \frac{d^2}{d\xi^2}, \dots . \tag{5}$$

Eq. (4) is then integrated as long as all terms contain derivatives where integration constants are considered zeros. Introducing a new independent variable

$$Y = \tanh(\xi), \tag{6}$$

leads to a change in the derivatives

$$\begin{gathered} \tfrac{d}{d\xi} = (1-Y^2)\tfrac{d}{dY}, \\ \tfrac{d^2}{d\xi^2} = (1-Y^2)\{-2Y\tfrac{d}{dY} + (1-Y^2)\tfrac{d^2}{dY^2}\}, \\ \tfrac{d^3}{d\xi^3} = (1-Y^2)\{(6Y^2-2)\tfrac{d}{dY} - 6Y(1-Y^2)\tfrac{d^2}{dY^2} + (1-Y^2)^2\tfrac{d^3}{dY^3}\}, \end{gathered} \tag{7}$$

and the remaining derivatives were derived similarly. The extended tanh method [19] admits the use of finite expansion

$$U(\xi) = S(Y) = a_0 + \sum_{k=1}^{m}[a_k Y^k + a_{-k}Y^{-k}], \tag{8}$$

where m is a positive integer which will be determined. The parameter m is usually obtained by balancing the highest order derivatives with the nonlinear terms in (4). Substituting (8) into (4) results in algebraic equations in powers of Y, that will lead to the determination of the parameters $a_k, (k = 0, 1, 2, 3, ...m)$, $a_{-k}, (k = 1, 2, 3, ..., m)$ and β.

3 The (2+1)-Dimensional Nizhnik-Novikov-Veselov Equations

In order to present some new types of the exact solutions to (1), we use the extended tanh method. On using the traveling wave transformations

$$\begin{aligned} u(x,y,t) = U(\xi) = a_0 + \textstyle\sum_{k=1}^{m}[a_k Y^k + a_{-k}Y^{-k}],\\ v(x,y,t) = V(\xi) = b_0 + \textstyle\sum_{k=1}^{n}[b_k Y^k + b_{-k}Y^{-k}],\\ w(x,y,t) = Z(\xi) = c_0 + \textstyle\sum_{k=1}^{l}[c_k Y^k + c_{-k}Y^{-k}], \end{aligned} \tag{9}$$

where $\xi = \alpha x + \lambda y - \beta t$, (1) becomes

$$\begin{gathered} -\beta U' + k\alpha^3 U''' + r\lambda^3 U''' + s\alpha U' + q\lambda U' - 3k\alpha(UV' + U'V) - 3r\lambda(UZ' + U'Z) = 0,\\ \alpha U' - \lambda V' = 0,\\ \lambda U' - \alpha Z' = 0. \end{gathered} \tag{10}$$

Balancing the U''' term with the UZ' in the first equation and U' term with V' or U' term with Z' in the third equation in (10) gives

$$m+3 = m+n+1, m+3 = m+l+1, m+1 = n+1, m+1 = l+1, \tag{11}$$

so that $m = n = l = 2$. The extended tanh method admits the use of the finite expansion

$$\begin{aligned} U(\xi) = a_0 + a_1 Y + a_2 Y^2 + \tfrac{a_{-1}}{Y} + \tfrac{a_{-2}}{Y^2},\\ V(\xi) = b_0 + b_1 Y + b_2 Y^2 + \tfrac{b_{-1}}{Y} + \tfrac{b_{-2}}{Y^2},\\ Z(\xi) = c_0 + c_1 Y + c_2 Y^2 + \tfrac{c_{-1}}{Y} + \tfrac{c_{-2}}{Y^2}. \end{aligned} \tag{12}$$

Substituting (12) into (10) and equating the coefficient of the powers of Y to zero, we obtain the following system of algebraic equations

$$\begin{aligned} 0 \;=\;& -\beta a_1 + 3a_1 b_0 \alpha k + 3a_0\, b_1 \alpha k + 2a_1 \alpha^3\, k - \beta a_{-1} + 3b_0 \alpha a_{-1} + 3b_2 \alpha k a_{-1}\\ &+2\alpha^3 k a_{-1} + 3a_0 \lambda c_1 r + 3b_1\, \alpha k a_{-2} + 3a_0 \alpha k b_{-1} + 3a_2 \alpha k b_{-1}\\ &+3a_1 \alpha k b_{-2} - a_1 \lambda q - \lambda a_{-1} q + 2a_1 \lambda^3 r + 3a_1 \lambda c_0 r\\ &+3a_0 \lambda c_{-1} r + 3a_2 \lambda c_{-1} r + 3a_1 \lambda c_{-2} r + 2\lambda^3 a_{-1} r + 3\lambda c_0 a_{-1} r \end{aligned}$$

$$
\begin{aligned}
& +3\lambda c_2 a_{-1} r + 3\lambda c_1 a_{-2} r - a_1 \alpha s - \alpha a_{-1} s, \\
0 &= 24\alpha^3 k a_{-2} - c_{-2}\alpha k a_{-2} b_{-2} + 24\lambda^3 a_{-2} r - 12\lambda c_{-2} a_{-2} r, \\
0 &= 6\alpha^3 k a_{-1} - 9\alpha k a_{-2} b_{-1} - 9\ \alpha k a_{-1} b_{-2} + 6\lambda^3 a_{-1} r - 9\lambda c_{-2} a_{-1} r - 9\lambda c_{-1} a_{-2} r, \\
0 &= 2\beta a_{-2} - 6 b_0\ \alpha k a_{-2} - 40\alpha^3 k a_{-2} - 6\alpha k a_{-1} b_{-1} - 6 a_0 \alpha k b_{-2} \\
& +c_{-2}\alpha k a_{-2} b_{-2} + 2\lambda a_{-2} q - 6 a_0 \lambda c_{-2} r - 6\lambda c_{-1} a_{-1} r - 40\lambda^3 a_{-2} r \\
& -6\lambda c_0 a_{-2} r + 12\lambda c_{-2} a_{-2} r + 2\alpha a_{-2} s, \\
0 &= \alpha a_{-1} - 3 b_0 \alpha k a_{-1} - 8\alpha^3 k a_{-1} - 3 b_1 \alpha k a_{-2} - 3 a_0 \alpha k b_{-1} + 9\alpha k a_{-2} b_{-1} \\
& -3 a_1 \alpha k b_{-2} + 9\alpha k a_{-1} b_{-2} + \lambda a_{-1} q - 3 a_0 \lambda c_{-1} r - 3 a_1 \lambda c_{-2} r \\
& -8\lambda^3 a_{-1} r - 3\lambda c_0 a_{-1} r + 9\lambda c_{-2} a_{-1} r - 3\lambda c_1 a_{-2} r + 9\lambda c_{-1} a_{-2} r + \alpha a_{-1} s, \\
0 &= -2\beta a_{-2} + 6 b_0 \alpha k a_{-2} + 16\alpha^3 k a_{-2} + 6\alpha k a_{-1} b_{-1} + 6 a_0 \alpha k b_{-2} - 2\lambda a_{-2} q \\
& +6 a_0 \lambda c_{-2} r + 6\lambda c_1 a_{-1} r + 16\lambda^3 a_{-2} r + 6\lambda c_0 a_{-2} r - 2\alpha a_{-2} s, \\
0 &= -2\beta a_2 + 6 a_2 b_0 \alpha k + 6 a_1 b_1 \alpha k + 6 a_0 b_2 \alpha k + 16 a_2 \alpha^3 k - 2 a_2 \lambda q \\
& +16 a_2 \lambda^3 r + 6 a_2 \lambda c_0 r + 6 a_1 \lambda c_1 r + 6 a_0 \lambda c_2 r - 2 a_2 \alpha s, \\
0 &= \beta c_{-1} - 3 a_1 b_0 \alpha k - 3 a_0 b_1 \alpha k + 9 a_2 b_1 \alpha k + 9 a_1 b_2 \alpha k - 8 a_1 \alpha^3 k \\
& -3 b_2 c_2 k a_{-1} - 3 a_2 b_{-1} + a_1 \lambda q - 8 a_1 \lambda^3 r - 3 a_1 \lambda c_0 r - 3 a_0 \lambda c_1 r \\
& +9 a_2 \lambda c_1 r + 9 a_1 \lambda c_2 r - 3 a_2 \lambda l_1 r - 3\lambda c_2 a_{-1} r + a_1 \alpha s, \\
0 &= 2\beta a_2 - 6 a_2 b_0 \alpha k - 6 a_1 b_1 \alpha k - 6 a_0 b_2 \alpha k + 12 a_2 b_2 \alpha k - 40 a_2 \alpha^3 k \\
& +2 a_2 \lambda q - 40 a_2 \lambda^3 r - 6 a_2 \lambda c_0 r - 6 a_1 \lambda c_1 r - 6 a_0 \lambda c_2 r + 12 a_2 \lambda c_2 r + 2 a_2 \alpha s, \\
0 &= -9 a_2 b_1 \alpha k - 9 a_1 b_2 \alpha k + 6 a_1 \alpha^3 k + 6 a_1 \lambda^3 r - 9 a_2 \lambda c_1 r - 9 a_1 \lambda c_2 r, \\
0 &= -12 a_2 b_2 \alpha k + 24 a_2 \alpha^3 k + 24 a_2 b^3 r - 12 a_2 \lambda c_2 r, -\lambda b_1 + a_1 \alpha + \alpha a_{-1} - \lambda b_{-1},
\end{aligned}
$$

$$
\begin{aligned}
0 &= -2\alpha a_{-2} + 2\lambda b_{-2}, \\
0 &= \alpha c_{-1} - \lambda a_{-1}, \\
0 &= -2\alpha c_{-2} + 2\lambda a_{-2}, \\
0 &= -2\lambda b_2 + 2 a_2 \alpha, \\
0 &= 2\lambda b_2 - 2 a_2 \alpha \\
0 &= -2 a_2 \lambda + 2\alpha c_2, \\
0 &= -a_1 \lambda + \alpha c_1, \\
0 &= \lambda b_1 - a_1 \alpha, \\
0 &= 2 a_2 \lambda - 2\alpha c_2, \\
0 &= -2 a_2 \lambda + 2\alpha c_2, \\
0 &= -a_1 \lambda + \alpha c_1, \\
0 &= \lambda b_1 - a_1 \alpha, \\
0 &= 2 a_2 \lambda - 2\alpha c_2, \\
0 &= -2\alpha c_{-2} + 2\lambda a_{-2} \\
0 &= \alpha c_{-1} - \lambda\ a_{-1}, \\
0 &= 2\alpha c_{-2} - 2\lambda a_{-2},
\end{aligned}
$$

$$0 = a_1\lambda - \alpha c_1 - \alpha c_{-1} + \lambda a_{-1},$$

These algebraic equations can be solved by Mathematica and give the following sets of solutions. The first set is

$$\begin{aligned}
&b_2 = b_{-2} = c_{-2} = c_2 = a_{-2} = a_2 = 0,\\
&b_0 = \frac{1}{3\alpha k}\{\beta + \lambda q + \alpha s - 3\lambda c_0 r\},\\
&b_{-1} = \frac{\lambda^2\ a_{-1} r}{k\alpha^2}, b_1 = \frac{a_1\lambda^2 r}{k\alpha^2}, c_1 = \frac{a_1\lambda}{\alpha}, c_{-1} = \frac{\lambda a_{-1}}{\alpha}.
\end{aligned}$$

The second set is

$$\begin{aligned}
&b_2 = b_{-2} = c_{-2} = c_2 = b_1 = c_1 = a_{-2} = a_2 = a_1 = 0,\\
&b_0 = \frac{1}{3\alpha k}\{\beta + \lambda q + \alpha s - 3\lambda c_0 r\},\\
&b_{-1} = \frac{\lambda^2 a_{-1} r}{\alpha^2 k}, c_{-1} = \frac{\lambda a_{-1}}{\alpha}.
\end{aligned}$$

The third set is

$$\begin{aligned}
&b_{-1} = b_1 = c_1 = c_{-1} = a_{-1} = a_1 = 0,\\
&b_0 = \frac{1}{\lambda\alpha^2 k(3\alpha^3 k + 3\lambda^3 r)}\{\beta\lambda\alpha^4\ k - 3a_0\alpha^6 k^2 - 8\lambda\alpha^7 k^2 + \lambda^2\alpha^4 kq\\
&+\beta\lambda^4\alpha r - 6\ a_0\ \lambda^3\alpha^3 kr - 16\lambda^4\alpha^4 kr - 3\lambda^2\alpha^4 c_0 kr + \lambda^5\alpha qr - 3a_0\ \lambda^6 r^2\\
&-8\lambda^7\alpha r^2 - 3\lambda^5\alpha c_0 r^2 + \lambda\alpha^5 ks + \lambda^4\alpha^2 rs\},\\
&b_{-2} = 2\alpha^2, b_2 = 2\alpha^2, c_2 = 2\lambda^2, c_{-2} = 2\lambda^2, a_2 = 2\alpha\lambda, a_{-2} = 2\alpha\lambda.
\end{aligned}$$

The fourth set is

$$\begin{aligned}
&b_2 = c_2 = b_{-1} = b_1 = c_1 = c_{-1} = 0 = a_2 = a_{-1} = a_1 = 0,\\
&b_0 = \frac{1}{\lambda\alpha^2 k(3\alpha^3 k + 3\lambda^3 r)}\{\beta\lambda\alpha^4\ k - 3a_0\alpha^6 k^2 - 8\lambda\alpha^7 k^2 + \lambda^2\alpha^4 kq\\
&+\beta\lambda^4\alpha r - 6\ a_0\ \lambda^3\alpha^3 kr - 16\lambda^4\alpha^4 kr - 3\lambda^2\alpha^4 c_0 kr + \lambda^5\alpha qr - 3a_0\ \lambda^6 r^2\\
&-8\lambda^7\alpha r^2 - 3\lambda^5\alpha c_0 r^2 + \lambda\alpha^5 ks + \lambda^4\alpha^2 rs\},\\
&b_{-2} = 2\alpha^2, c_{-2} = 2\lambda^2, a_{-2} = 2\alpha\lambda.
\end{aligned}$$

The fifth set is

$$\begin{aligned}
&b_{-2} = c_{-2} = b_{-1} = b_1 = c_1 = c_{-1} = a_{-2} = a_{-1} = a_1 = 0,\\
&b_0 = \frac{1}{\lambda\alpha^2 k(3\alpha^3 k + 3\lambda^3 r)}\{\beta\lambda\alpha^4\ k - 3a_0\alpha^6 k^2 - 8\lambda\alpha^7 k^2 + \lambda^2\alpha^4 kq\\
&+\beta\lambda^4\alpha r - 6\ a_0\ \lambda^3\alpha^3 kr - 16\lambda^4\alpha^4 kr - 3\lambda^2\alpha^4 c_0 kr + \lambda^5\alpha qr - 3a_0\ \lambda^6 r^2\\
&-8\lambda^7\alpha r^2 - 3\lambda^5\alpha c_0 r^2 + \lambda\alpha^5 ks + \lambda^4\alpha^2 rs\},\\
&b_2 = 2\alpha^2, c_2 = 2\lambda^2, a_2 = 2\lambda\alpha.
\end{aligned}$$

In view of these we obtain the following kinds of solutions

$$u_1(x,y,t) = a_0 + a_1 \tanh\xi + a_{-1}\coth\xi,$$
$$v_1(x,y,t) = \frac{1}{3\alpha k}\{\beta + \lambda q + \alpha s - 3\lambda c_0 r\} + \frac{a_1\lambda^2 r}{k\alpha^2}\tanh\xi + \frac{\lambda^2\ a_{-1} r}{k\alpha^2}\coth\xi,$$
$$w_1(x,y,t) = c_0 + \frac{a_1\lambda}{\alpha}\tanh\xi + \frac{\lambda a_{-1}}{\alpha}\coth\xi,$$

$$u_2(x,y,t) = a_{-1}\coth\xi,$$
$$v_2(x,y,t) = \frac{1}{3\alpha k}\{\beta + \lambda q + \alpha s - 3\lambda c_0 r\} + \frac{\lambda^2 a_{-1} r}{\alpha^2 k}\coth\xi,$$
$$w_2(x,y,t) = c_0 + \frac{\lambda a_{-1}}{\alpha}\coth\xi,$$

$$u_3(x,y,t) = a_0 + 2\lambda\alpha\{\tanh^2\xi + \coth^2\xi\},$$
$$v_3(x,y,t) = b_0 + 2\alpha^2\{\tanh^2\xi + \coth^2\xi\},$$
$$w_3(x,y,t) = c_0 + 2\lambda^2\{\tanh^2\xi + \coth^2\xi\},$$

$$u_4(x,y,t) = a_0 + 2\lambda\alpha\coth^2\xi,$$
$$v_4(x,y,t) = b_0 + 2\alpha^2\coth^2\xi\,,$$
$$w_4(x,y,t) = c_0 + 2\lambda^2\coth^2\xi,$$

and

$$u_5(x,y,t) = a_0 + 2\lambda\alpha\tanh^2\xi,$$
$$v_5(x,y,t) = b_0 + 2\alpha^2\tanh^2\xi,$$
$$w_5(x,y,t) = c_0 + 2\lambda^2\tanh^2\xi,$$

where $\xi = \alpha x + \lambda y - \beta t$, a_0, a_1, a_{-1} and c_0 are arbitrary constants, b_0 defined in the fifth set.

4 The (1+1)-Dimensional Jaulent-Miodek (JM) Equations

In this section, we will use the extended tanh method to handle (2). Let

$$\begin{aligned} u(x,t) = U(\xi) = a_0 + \textstyle\sum_{k=1}^{m}[a_k Y^k + a_{-k}Y^{-k}],\\ v(x,t) = V(\xi) = b_0 + \textstyle\sum_{k=1}^{n}[b_k Y^k + b_{-k}Y^{-k}], \end{aligned} \tag{13}$$

where $\xi = \alpha(x + \beta t)$. Then (2) becomes

$$\begin{aligned} \alpha\beta U' + \alpha^3 U''' + \tfrac{3\alpha^3}{2}VV''' + \tfrac{9\alpha^3}{2}V'V'' - 6\alpha UU' - 6\alpha UVV' - \tfrac{3\alpha}{2}U'V^2 = 0,\\ \alpha\beta V' + \alpha^3 V''' - 6\alpha U'V - 6\alpha UV' - \tfrac{15\alpha}{2}V'V^2 = 0. \end{aligned} \tag{14}$$

Balancing the highest derivatives term with highest nonlinear terms in (14) gives

$$m+3=2n+3 \Rightarrow m=2n, n+3=3n+1, \tag{15}$$

so that $m=2, n=1$. The extended tanh method admits the use of the finite expansion

$$\begin{aligned} U(\xi) &= a_0 + a_1 Y + a_2 Y^2 + \tfrac{a_{-1}}{Y} + \tfrac{a_{-2}}{Y^2}, \\ V(\xi) &= b_0 + b_1 Y + \tfrac{b_{-1}}{Y}. \end{aligned} \tag{16}$$

Substituting (16) into (14) and equating the coefficient of the powers of Y to zero, we obtain the following system of algebraic equations

$$\begin{aligned}
0 &= \beta a_1\alpha - 6a_0a_1\alpha - \frac{3}{2}a_1b_0^2\alpha - 6a_0b_0b_1\alpha - 2a_1\alpha^3 - 3b_0b_1\alpha^3 + \beta\alpha a_{-1} - 6a_0\alpha a_{-1} \\
&\quad -6a_2\alpha a_{-1} - \frac{3}{2}b_0^2\alpha a_{-1} - \frac{9}{2}b_1^2\alpha a_{-1} - 2\alpha^3 a_{-1} - 6a_1\alpha a_{-2} - 6a_0b_0\alpha b_{-1} \\
&\quad -3a_1b_1\alpha b_{-1} - 3b_0\alpha^3 b_{-1} - 3b_1\alpha a_{-1}b_{-1} - \frac{9}{2}a_1\alpha b_{-1}^2 \\
0 &= -24\alpha^3 a_{-2} + 12\alpha a_{-2}^2 - 18\alpha^3 b_{-1}^2 + 9\alpha a_{-2}b_{-1}^2, \\
0 &= -6\alpha^3\ a_{-1} + 18\alpha a_{-1}a_{-2} - 9b_0\alpha^3 b_{-1} + 12b_0\alpha a_{-2}b_{-1} + \frac{15\alpha a_{-1}b_{-1}^2}{2}, \\
0 &= 6\alpha a_{-1}^2 - 2a_1\alpha a_{-2} + 12a_0\alpha a_{-2} + 3b_0^2\alpha a_{-2} + 40\alpha^3 a_{-2} - 12\alpha a_{-2}^2 \\
&\quad +9b_0\alpha a_{-1}b_{-1} + 6b_1\alpha a_{-2}b_{-1} + 6a_0\alpha b_{-1}^2 + 30\ \alpha^3 b_{-1}^2 - 9\alpha a_{-2}b_{-1}^2, \\
0 &= -\beta\alpha a_{-1} + 6\ a_0\alpha a_{-1} + \frac{3b_0^2\alpha a_{-1}}{2} + 8\alpha^3\ a_{-1} + 6a_1\alpha a_{-2} - 18\alpha a_{-1}a_{-2} + 6a_0b_0\alpha b_{-1} \\
&\quad +12b_0\ \alpha^3 b_{-1} + 3b_1\alpha a_{-1}b_{-1} - 12\ b_0\ \alpha a_{-2}b_{-1} + \frac{9a_1\alpha b_{-1}^2}{2} - \frac{15\alpha a_{-1}b_{-1}^2}{2}, \\
0 &= -3b_0b_1\alpha a_{-1} - 6\alpha a_{-1}^2 + 2\beta\alpha a_{-2} - 12a_0\alpha a_{-2} - 3\ b_0^2\alpha a_{-2} - 3\ b_1^2\alpha a_{-2} - 16\ \alpha^3 a_{-2} \\
&\quad +3\ a_1b_0\alpha b_{-1} - 9\ b_0\alpha a_{-1}b_{-1} - 6b_1\alpha a_{-2}b_{-1} - 6\ a_0\alpha b_{-1}^2 + 3\ a_2\alpha b_{-1}^2 - 12\alpha^3 b_{-1}^2, \\
0 &= -6a_1^2\alpha + 2\beta a_2\alpha - 12\ a_0\ a_2\alpha - 3\ a_2\ b_0^2\alpha - 9\ a_1\ b_0b_1\alpha - 6\ a_0b_1^2\alpha - 16\ a_2\alpha^3 \\
&\quad -12\ b_1^2\alpha^3 + 3\ b_2\ b_1\alpha a_{-1} + 3b_1^2\alpha a_{-2} - 3a_1b_0\alpha b_{-1} - 6a_2b_1\alpha b_{-1} - 3a_2\alpha b_{-1}^2, \\
0 &= -\beta a_1\alpha + 6\ a_0a_1\alpha - 18a_1a_2\alpha + \frac{3}{2}a_1b_0^2\alpha + 6a_0b_0b_1\alpha - 12a_2b_0b_1\alpha \\
&\quad -\frac{15}{2}a_1\ b_1^2\alpha + 8a_1\alpha^3 + 12b_0b_1\alpha^3 + 6a_2\alpha a_{-1} + \frac{9}{2}b_1^2\alpha a_{-1} + 3a_1b_1\alpha b_{-1}, \\
0 &= 6a_1^2\alpha - 2\beta a_2\alpha + 12a_0a_2\alpha - 12a_2^2\alpha + 3a_2\ b_0^2\alpha + 9a_1b_0b_1\alpha \\
&\quad +6\ a_0b_1^2\alpha - 9\ a_2b_1^2\alpha + 40a_2\alpha^3 + 30\ b_1^2\alpha^3 + 6a_2b_1\alpha b_{-1}, \\
0 &= 18a_1a_2\alpha + 12a_2b_0b_1\alpha + \frac{15}{2}a_1b_1^2\alpha - 6a_1\alpha^3 - 9b_0b_1\alpha^3, \\
0 &= 12a_2^2\alpha + 9a_2b_1^2\alpha - 24a_2\alpha^3 - 18b_1^2\alpha^3, \\
0 &= -6a_1b_0\alpha + \beta b_1\alpha - 6a_0b_1\alpha - \frac{15}{2}b_0^2\ b_1\alpha - 2b_1\alpha^3 - 6b_0\alpha a_{-1} - 6\ b_1\alpha a_{-2} + \alpha\beta b_{-1} \\
&\quad -6a_0\alpha b_{-1} - 6a_2\alpha b_{-1} - \frac{15}{2}b_0^2\alpha b_{-1} - \frac{15}{2}b_1^2\alpha b_{-1} - 2\alpha^3 b_{-1} - \frac{15}{2}b_1\alpha b_{-1}^2,
\end{aligned}$$

$$
\begin{aligned}
0 &= -6\alpha^3 b_{-1} + 18\alpha a_{-2} b_{-1} + \frac{15\alpha b_{-1}^3}{2}, \\
0 &= 12 b_0 \alpha a_{-2} + 12\alpha a_{-1} b_{-1} + 15 b_0 \alpha b_{-1}^2 = 0, 6 b_0 \alpha a_{-1} + 6 b_1 \alpha a_{-2} - \beta\alpha b_{-1} + 6 a_0 \alpha b_{-1} \\
&\quad + \frac{15 b_0^2 \alpha b_{-1}}{2} + 8\alpha^3\ b_{-1} - 18\alpha b_{-2} b_{-1} + \frac{15 b_1 \alpha b_{-1}^2}{2} - \frac{15\alpha b_{-1}^3}{2}, \\
0 &= -12 b_0 \alpha a_{-2} - 12\alpha a_{-1} b_{-1} - 15 b_0 \alpha b_{-1}^2 = 0, -12 a_2\ b_0 \alpha - 12\ a_1 b_1 \alpha - -15 b_0 b_1^2 \alpha, \\
0 &= 6 a_1 b_0 \alpha - \alpha b_1 \alpha + 6 a_0 b_1 \alpha - 18 a_{21} \alpha + \frac{15}{2} b_0^2\ b_1 \alpha - \frac{15}{2} b_1^3 \alpha \\
&\quad +8\ b_1 \alpha^3 + 6 a_2 \alpha b_{-1} + \frac{15}{2} b_1^2 \alpha b_{-1}, \\
0 &= 12 a_2 b_0 \alpha + 12 a_1 b_1 \alpha + 15 b_0 b_1^2 \alpha, \\
0 &= 18 a_2 b_1 \alpha + \frac{15}{2} b_1^3 \alpha - 6 b_1 \alpha^3.
\end{aligned}
$$

These algebraic equations can be solved by Mathematica to yield the following sets of solutions. The first set is

$$a_1 = a_{-1} = b_0 = a_0 = 0, a_2 = a_{-2} = 2\alpha^2, b_1 = b_{-1} = -2i\alpha, \beta = -16\alpha^2.$$

The second set is

$$a_1 = a_{-1} = b_0 = b_1 = a_2 = 0, a_{-2} = 2\alpha^2, b_{-1} = -2i\alpha, a_0 = -\alpha^2, \beta = -4\alpha^2.$$

The third set is

$$b_0 = a_1 = a_{-1} = 0, b_1 = b_{-1} = -i\alpha, = a_0 = \frac{\alpha^2}{2}, a_2 = a_{-2} = \frac{3\alpha}{4}, \beta = -4\alpha^2.$$

The fourth set is

$$
\begin{aligned}
&a_{-1} = a_{-2} = b_{-1} = 0, a_0 = \frac{-1}{2}(\frac{b_0^2}{2} + \alpha^2), a_1 = \frac{i b_0 \alpha}{2}, b_1 = -i\alpha, a_2 = \frac{3\alpha}{4}, \\
&\beta = \frac{1}{2}(6 b_0^2 - 2\alpha^2).
\end{aligned}
$$

The fifth set is

$$b_0 = a_1 = a_{-1} = 0, a_0 = -2\alpha^2, a_2 = a_{-2} = 2\alpha^2, b_1 = -2i\alpha, b_{-1} = 2i\alpha, \beta = 8\alpha^2.$$

In view of these we obtain the following kinds of solutions

$$
\begin{aligned}
u_1(x,t) &= 2\alpha^2\{\tanh^2(\alpha[x - 16\alpha^2 t]) + \coth^2(\alpha[x - 16\alpha^2 t])\}, \\
v_1(x,t) &= -2i\alpha\{(\tanh(\alpha[x - 16\alpha^2 t]) + \coth(\alpha[x - 16\alpha^2 t])\},
\end{aligned}
$$

$$
\begin{aligned}
u_2(x,t) &= -\alpha^2 + 2\alpha^2 \coth^2(\alpha[x - 4\alpha^2 t]), \\
v_2(x,t) &= -2i\alpha \coth(\alpha[x - 4\alpha^2 t]),
\end{aligned}
$$

$$u_3(x,t) = \frac{\alpha^2}{2} + \frac{3\alpha}{4}\{\tanh^2(\alpha[x-4\alpha^2 t]) + \coth^2(\alpha[x-4\alpha^2 t])\},$$
$$v_3(x,t) = -i\alpha\{(\tanh(\alpha[x-4\alpha^2 t]) + \coth(\alpha[x-4\alpha^2 t])\},$$

$$\begin{aligned} u_4(x,t) &= \frac{-1}{2}(\frac{b_0^2}{2} + \alpha^2) + \frac{ib_0\alpha}{2}\tanh(\alpha[x+\frac{1}{2}(6b_0^2 - 2\alpha^2)t]) \\ &\quad + \frac{3\alpha}{4}\tanh^2(\alpha[x+\frac{1}{2}(6b_0^2 - 2\alpha^2)t]), \\ v_4(x,t) &= b_0 - i\alpha\tanh(\alpha[x+\frac{1}{2}(6b_0^2 - 2\alpha^2)t]), \end{aligned}$$

and

$$u_5(x,t) = -2\alpha^2 + 2\alpha^2\{\tanh^2(\alpha[x+8\alpha^2 t]) + \coth^2(\alpha[x+8\alpha^2 t])\},$$
$$v_5(x,t) = -2i\alpha\{(\tanh(\alpha[x+8\alpha^2 t]) - \coth(\alpha[x+8\alpha^2 t])\},$$

where b_0 is arbitrary constant.

5 Conclusions

In this article, the extended tanh method was applied to give the traveling wave solutions of the coupled (2+1)-dimensional Nizhnik-Novikov-Veselov and the (1+1)-dimensional Jaulent-Miodek (JM) equations. The extended tanh method was successfully used to establish these solutions. Many well know nonlinear wave equations were handled by this method to show the new solutions compared to the solutions obtained in [4, 16]. The performance of the extended tanh method is reliable and effective and gives more solutions. The applied method will be used in further works to establish entirely new solutions for other kinds of nonlinear wave equations.

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References

[1] H. Segur and M. J. Ablowitz, Solitons and Inverse Scattering Transform, Philadelphia: SIAM; 1981.

[2] F. Pempinelli, M. Manna, J. J. P. Leon and M. Boiti, On the spectral transform of the Korteweg-de Vries equation in two spatial dimensional, Inverse Probl., 2(1986), 271–279.

[3] M. A. Abdou and S. A. El-Wakil, New exact traveling wave solutions using modified extended tanh function method, Chaos Soliton Fractal, 31(2007), 840–852.

[4] E. G. Fan, Uniformly constructing a series of explicit exact solutions to nonlinear equations in mathematical physics, Chaos Solitons Fractals, 16(2003), 819–839.

[5] H. Zhang and E. Fan, A note on the homogeneous balance method, Phys. Lett. A , 264(1998), 403–406.

[6] E. Fan, Extended tanh function method and its applications to nonlinear equations, Phys. Lett. A, 277(2002), 212–218.

[7] R. Hirota, Direct method of finding exact solutions of nonlinear evolution equations, In: Bullough R, Caudrey P, editors. Bäcklund tansformations, Berlin: Springer; (1980) 1157–1175.

[8] K. Miodek and M. Jaulent, Nonlinear evolution equations associated with energy dependent Schrodinger potentials, Lett. Math. Phys., 1(1976), 243–250.

[9] S. Y. Lou, On the coherent structures of the Nizhnik Novikov Veselov equation, Phys. Lett. A, 277(2000), 94–100.

[10] W. Malfliet, Solitory wave solutions of nonlinear wave equations, Am. J. Phys., 60(1992), 650–654.

[11] W. Hereman and W. Malfliet, The tanh method I: exact solutions of nonlinear evolution and wave equations, Phys. Scripta, 54(1996) , 563–568.

[12] Y. Matsuna, Reduction of dispersionless coupled KdV equations to the Euler-Darboux equation, J. Math. Phys., 42(2001), 1744–1760.

[13] A. J. Morrison, E. J. Parkes and V. O. Vakhnenko , A Bäcklund transformation and the inverse scattering transform method for the generalised Vakhnenko equation, Chaos Soliton Fractal, 17(2003), 683–692.

[14] H. Q. Zhang and Y. J. Ren, A generalized F-expansion method to find abundant families of Jacobi Ellipitic function solutions of the (2+1)-dimensional Nizhnik Novikov Veselov, Chaos Solitons Fractal, 27(2006), 959–979.

[15] Z. Yan, Abundant families of Jacobi elliptic function solutions of the (2+1)-dimensional integrable Davey-Stewartson-type equation via a new method, Chaos Soliton Fractal, 18(2003), 299–309.

[16] A. Bakir and E. Yusufoglu, Exact solutions of nonlinear evolution equations, Chaos Solitons Fractals, 37(2008), 842–848.

[17] A. M. Wazwaz, The tanh method for traveling wave solutions of nonlinear equations, Appl. Math. Comput., 154(2004), 713–723.

[18] A. M. Wazwaz, Traveling wave solutions of generalized forms of Burger, Burger-KdV and Burger-Huxley equations, Appl. Math. Comput.,169(2005), 639–656.

[19] A. M. Wazwaz, The tanh method: exact solutions of the Sine-Gordon and Sinh-Gordon equations, Appl. Math. Comput., 167(2005), 1196–1210.

[20] A. M. Wazwaz, The tanh method: solitons and periodic solutions for the Dodd-Bullough-Mikhailov and the Tzitzeica-Dodd-Bullough equations, Chaos Soliton Fractal, 25(2005), 55–63.

[21] A. M. Wazwaz, The extended tanh method for new soliton solutions for many forms of the fifth-order KdV equations, Appli. Math. Comput. 184(2007), 1002–1014.

[22] A. M. Wazwaz, New solitary wave solutions to modified forms of Degasperis-Procesi and Camassa-Holm equations, Appl. Math. Comput., 186(2007), 130–141.

[23] H. Q. Zhang, B. Li and T. C. Xia, new explicit and exact solutions for the Nizhnik-Novikov-Veselov equation, Appl. Math. E. Notes, 1(2001), 139–142.

[24] Y. P. Zheng, Separability and dynamical r-matrix for the constrainted flows of the Jaulent Miodek hierarchy, Phys. Lett. A, 216(1996), 26–32.

[25] R. G. Zhou, The finite band solution of the Jaulent Miodek equation, J. Math. Phys. 38(1999), 2535–2546.