1. If \{f_n\} is a sequence of continuous functions on \([0, 1]\) such that \(0 \leq f_n \leq 1\) and such that \(f_n(x) \to 0\) as \(n \to \infty\) for every \(x \in [0, 1]\), show that

\[
\lim_{n \to \infty} \int_0^1 f_n(x) \, dx = 0.
\]

2. Suppose \(\phi\) is a real function on \(\mathbb{R}\) such that

\[
\phi(\int_0^1 f(x) \, dx) \leq \int_0^1 \phi(f) \, dx
\]

for every real bounded measurable \(f\). Prove that \(\phi\) is convex.

3. Let \(X\) be a normed linear space and let \(X^*\) be its dual space with the norm

\[
\|f\| = \sup\{|f(x)| : \|x\| \leq 1\}.
\]

(a) Prove that \(X^*\) is a Banach space.

(b) Prove that the mapping \(f \to f(x)\) is, for each \(x \in X\), a bounded linear functional on \(X^*\), of norm \(\|x\|\).

(c) Prove that \(\{\|x_n\|\}\) is bounded if \(\{x_n\}\) is a sequence in \(X\) such that \(\{f(x_n)\}\) is bounded for every \(f \in X^*\).

4. Evaluate the following integral:

\[
\int_{-\infty}^{\infty} \frac{3x^2 + 2}{(x^2 + 4)(x^2 + 9)} \, dx.
\]

5. Let \(\Omega\) be the upper half of the unit disc \(U\). Find the conformal mapping \(f\) of \(\Omega\) onto \(U\) that carries \([-1, 0, 1]\) to \([-1, -i, 1]\). Find \(z \in \Omega\) such that \(f(z) = 0\). Find \(f(i/2)\).