Ph.D. Qualifying Examination
Matrix Theory
Sep. 13, 2007

Please write down all the detail of your computation and answers.

1. (15%) Find the similarity transformation to convert the matrix

\[
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 3 & 0
\end{pmatrix}
\]

to its Jordan canonical form.

2. (15%) Find permutation matrix \( P \), lower triangular matrix \( L \) and upper triangular matrix \( U \) such that

\[
\begin{pmatrix}
0 & 0 & -1 & 1 & 1 \\
0 & 0 & 1 & -1 & 0 \\
0 & -1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 1
\end{pmatrix} = PLU.
\]

3. (15%) Let \( m \times n \) matrix \( A \) be the matrix representation of linear transformation \( L \). Show that

(1) \( L \) is one-to-one \( \iff \) all columns of \( A \) are linearly independent,
(2) \( L \) is onto \( \iff \) all rows of \( A \) are linearly independent.

4. (15%) Prove that a complex square matrix is unitarily diagonalizable \( \iff \) it is normal.

5. (15%) Show that for \( n \times n \) matrix \( A \), the induced matrix 2-norm can be computed by

\[
\|A\|_2 = \sigma_1(A) = \sqrt{\rho(AA^*)} = \sqrt{\rho(A^*A)},
\]

where \( \sigma_1(A) \) and \( \rho(A) \) are the largest singular value and the spectral radius of \( A \), respectively.

6. (15%) Let \( n \times n \) matrix \( A \) have all entries 1. Find all of its eigenvalues and corresponding eigenvectors.

7. (10%) Find the projection matrix onto the plane \( x - y + z = 0 \) in \( \mathbb{R}^3 \).