1. Prove or disprove:

   (a) if $p$ is a prime, then $p$ is a divisor of $(p - 1)! + 1$.
   
   (b) if $p$ is a divisor of $(p - 1)! + 1$, then $p$ is a prime.

2. Classify all the groups which has order $10$. And prove it.

3. Prove or disprove: if $R$ is a finite integral domain, then $R$ is a field.

4. Let $R$ be a commutative ring and $f(x) = a_0 + a_1x + \cdots + a_n x^n \in R[x]$. Prove or disprove: if $f(x)$ is nilpotent, then $a_i$ is nilpotent for all $i = 0, 1, \ldots, n$.

5. Let $\alpha_1, \alpha_2, \ldots, \alpha_t$ be roots of $f(X) \in \mathbb{Q}[X]$. Prove or disprove:

\[ \mathbb{Q}(\alpha_1, \alpha_2, \ldots, \alpha_t) = \mathbb{Q}[\alpha_1, \alpha_2, \ldots, \alpha_t] \]

6. Find a polynomial $f(X) \in \mathbb{Q}[X]$ with degree $3$ such that the Galois group of $\mathbb{Q}(\alpha_1, \alpha_2, \alpha_3)$ over $\mathbb{Q}$ is isomorphism to $\mathbb{Z}_3$ (where $\alpha_1, \alpha_2, \alpha_3$ are the three roots of $f(X)$). And prove it (i.e. prove the polynomial you give satisfy the desired properties).

1, 2: 20 points; 3–6: 15 points.