

# Numerical PDEs Day with Jim Douglas's Family

Prof. Douglas wrote over 200 papers with over 70 co-authors. Among his many recognitions were the Cedric K. Ferguson Medal from the Society of Petroleum Engineers (1958), the Robert Earll McConnell Award from the American Institute of Mining, Metallurgical, and Petroleum Engineers (1979), and a Commemorative Medal from Charles University, Prague (1992). Jim was a Fellow of SIAM and in the inaugural class of Fellows of the American Mathematical Society.

He had a tremendous influence on young people, both as an advisor to graduate students and as someone who helped shape many professional mathematicians early in their career. One can count among his students and post-doctoral associates many Fellows of the AMS and SIAM and leaders in computational science both in the US and abroad.

We are happy to have 4 members from his family to attend this Numerical PDEs day.

日期：108年6月20日(四)下午3:10~6:00

地點：中山大學理學院 SC4009-1 室

## 議程

時間	議題	主講人
15:00-15:10	Opening	Chieh-Sen Huang
15:10-15:45	P1-nonconforming polyhedral finite elements in high dimensions	Dongwoo Sheen Department of Mathematics Seoul National University
15:50-16:25	Regularity for elliptic equations in fibre-reinforced composites	Li-Ming Yeh Department of Applied Mathematics, NCTU
16:30-17:20	Mixed finite element methods for second order elliptic problems (I)	Todd Arbogast University of Texas at Austin, USA
17:20-18:00	Mixed finite element methods for second order elliptic problems (II)	Todd Arbogast University of Texas at Austin, USA
18:30~	晚餐	

## **Title : $P_1$ -nonconforming polyhedral finite elements in high dimensions**

Dongwoo Sheen  
Department of Mathematics  
Seoul National University

### **Abstract**

We consider the lowest-degree nonconforming finite element methods for the approximation of elliptic problems in high dimensions. The  $P_1$ -nonconforming polyhedral finite element is introduced for any high dimension. Our finite element is simple and cheap as it is based on the triangulation of domains into polytopes, which are combinatorially equivalent to the  $d$ -dimensional unit cube, rather than the triangulation of domains into simplices. Our nonconforming element is nonparametric, and on each polytope it contains only linear polynomials, but it is sufficient to give optimal order convergence for second-order elliptic problems.

## **Title : Regularity for elliptic equations in fibre-reinforced composites**

Li-Ming Yeh  
Department of Applied Mathematics, NCTU

### **Abstract**

Some results on the regularity for the solutions of the elliptic equations in fibre-reinforced composites are presented. The composites consist of an  $\varepsilon$ -periodic lattice of reinforced fibres with high conductivity, included in a connected matrix with low conductivity. The coefficients of the elliptic equations, depending on the conductivities, are equi-coercive and not bounded above. The elliptic solutions have fast diffusion in the reinforced fibres and slow diffusion in the connected matrix. Let  $\omega^2 \in [1, \infty)$  denote the conductivity ratio of the reinforced fibres to the connected matrix and let  $\varepsilon, \mu/2 \in (0, 1)$  denote the diameter of each fibre. In this talk, Hölder and gradient  $L^q$  estimates uniform in  $\varepsilon, \omega, \mu$  for the elliptic solutions in the whole composites are derived. In these results, the volume fraction of the fibres is independent of the periodic size  $\varepsilon$ . Because of the high conductivity in the reinforced fibres, it is shown that the elliptic solutions are always smoother in the reinforced fibres than in the connected matrix.

## Title : Mixed finite element methods for second order elliptic problems

Todd James Arbogast  
University of Texas at Austin, USA

### Abstract

Second order elliptic equations  $-\text{div}(a \text{ grad } p) = f$  can be solved approximately for the scalar variable  $p$  directly. However the vector flux  $u = -a \text{ grad } p$  is often the variable of interest. Mixed methods write the equation as a system of first order equations for  $p$  and  $u$ . Finite element solution provides accurate approximation of both variables. We describe the basic theory of mixed methods, including the need for an inf-sup condition and implementation in the hybrid form. We review existing families of mixed finite elements and discuss new families of finite elements that work well on quadrilaterals and cuboidal hexahedra. To show the richness of mixed method approaches, we then discuss several applications, including two-phase flow in a porous medium, variational multiscale techniques (i.e., multiscale finite elements), multiscale mortar methods, and mantle dynamics (as time permits).

中山大學應用數學系  
敬請公告！歡迎參加！

應用數學系：<http://math.nsysu.edu.tw>

校園地圖：[http://math.nsysu.edu.tw/var/file/183/1183/img/779/nsysu\\_math\\_map.jpg](http://math.nsysu.edu.tw/var/file/183/1183/img/779/nsysu_math_map.jpg)

交通資訊：<https://www.nsysu.edu.tw/p/412-1000-4132.php?Lang=zh-tw>



應用數學系



校園地圖



交通資訊