國立中山大學應用數學系
學術演講

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講　　題：Three talks concerning nonlinear boundary value
　　　　　problems

時　　間：2018/07/12（星期四）14:10～17:00

地　　點：理學院四樓理 SC 4009-1 室

茶　　會：15:00～ 15:10, 16:00 ～ 16:10 於理 SC 4010
　　　　　室（系辦公室）

中山大學應用數學系

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應用數學系：http://math.nsysu.edu.tw
校園地圖：http://math.nsysu.edu.tw/ezfiles/87/1087/img/779/NSYSUMAPmath990705.jpg
交通資訊：http://www.nsysu.edu.tw/files/11-1000-4132.php

應用數學系　　　　　校園地圖　　　　　交通資訊

Three talks concerning nonlinear boundary value problems presented at Seminar in National Sun Yat-sen University

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1   Unified approach to some boundary value problems for fully fourth order differential equations

In this talk we present new results concerning a unified approach to investigate boundary value problems for fully fourth order nonlinear differential equation

$$u^{(4)}(x) = f(x, u(x), u'(x), u''(x), u'''(x)), \quad 0 < x < 1, \qquad (1.1)$$

$$
\begin{aligned}
B_1(\bar{u}) = 0, \ B_2(\bar{u}) = 0, \\
B_3(\bar{\bar{u}}) = 0, \ B_4(\bar{\bar{u}}) = 0,
\end{aligned}
\qquad (1.2)
$$

where B_1, B_2, B_3, B_4 are linear combinations of the components of $\bar{u}$ and $\bar{\bar{u}}$,

$$
\begin{aligned}
\bar{u} &= \big(u(0), u(1), u'(0), u'(1)\big), \\
\bar{\bar{u}} &= \big(u''(0), u''(1), u'''(0), u'''(1)\big).
\end{aligned}
$$

The problems (3.2), (2.2) include as particular cases many problems considered before by other authors using different methods. Our method is based

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on the reduction of the problems to operator equations for the nonlinear terms but not for functions to be sought. By this approach we establish the existence, uniqueness and positivity of solutions of the problems under the conditions which are much simpler and weaker than those in known works of the other authors. The theoretical results are illustrated on examples.

2 A novel approach to the Dirichlet problem for fully fourth order differential equations

In this talk, after reviewing some our recent results in studying some problems for fourth order nonlinear equations, we present new results of the existence and uniqueness of a solution and propose an iterative method for solving a beam problem which is described by the fully fourth order equation associated with the Dirichlet boundary conditions

$$u^{(4)}(x) = f(x, u(x), u'(x), u''(x), u'''(x)), \quad a < x < b,$$
$$u(a) = A_1, \quad u(b) = B_1, \quad u'(a) = A_2, \quad u'(b) = B_2,$$

where $f \in C[[a,b] \times \mathbb{R}^4 \to \mathbb{R}]$, A_i, B_i $(i = 1, 2)$ are real constants. This problem was well studied by Agarwal by the reduction of it to a nonlinear operator equation for the unknown function $u(x)$. Here we propose a novel approach by the reduction of the problem to an operator equation for the nonlinear term $\varphi(x) = f(x, u(x), u'(x), u''(x), u'''(x))$. Under some easily verified conditions on the function f in a specified bounded domain, we prove the existence, uniqueness and positivity of a solution and the convergence of an iterative method for finding it. Some examples demonstrate the applicability of the theoretical results and the efficiency of the iterative method. The advantages of the proposed approach to the problem over the well-known approach of Agarwal is shown on examples.

3 An efficient method for solving nonlinear biharmonic problems

In this talk we are concerned with the solvability and numerical solution of two problems for nonlinear biharmonic problems:

Problem 1. Consider the following nonlinear biharmonic boundary value problem (BVP)

$$\Delta^2 u = f(x, u, \Delta u), \quad x \in \Omega,$$
$$u = 0, \quad \Delta u = 0, \quad x \in \Gamma, \tag{3.1}$$

where Ω is a connected bounded domain in $\mathbb{R}^2$, with a smooth boundary Γ, Δ is the Laplace operator, $\partial/\partial\nu$ the outward normal derivative on Γ. We assume that $f(x, u, v)$ is a function continuous in a bounded domain, which will be indicated later. The problem (3.1) describes the static deflection of an elastic bending plate with hinged edges rested on nonlinear foundation.

We propose the reduction of the problem to finding fixed point of a nonlinear operator for the nonlinear term. The result is that under some easily verified conditions we have established the existence and uniqueness of a solution and the convergence of an iterative method for the solution. The positivity of the solution and the monotony of iterations are also considered. Some examples demonstrate the applicability of the obtained theoretical results and the efficiency of the iterative method.

Problem 2. Consider the boundary value problem for the nonlinear biharmonic equation of Kirchhoff type

$$\Delta^2 u = M\left(\int_\Omega |\nabla u|^2 dx\right)\Delta u + f(x, u), \quad x \in \Omega,$$
$$u = 0, \quad \Delta u = 0, \quad x \in \Gamma, \tag{3.2}$$

where Ω is a connected bounded domain in $\mathbb{R}^K$ ($K \geq 2$) with a smooth boundary Γ, Δ is the Laplace operator, Δ^2 is the biharmonic operator, ∇u is the gradient of u, $f : \Omega \times \mathbb{R} \to \mathbb{R}$ and $M : \mathbb{R}^+ \to \mathbb{R}$ are continuous functions. For investigating the above problem we propose an efficient approach by the reduction of the problem to an operator equation for the nonlinear part of the equation. The result is that we have established the existence and uniqueness of a solution under some easily verified conditions. Also, we propose an iterative method for finding the solution. Some examples demonstrate the applicability of the theoretical results and the efficiency of the iterative method.