科目：微積分【應數系二年級】　　　　共 1 頁第 1 頁

1－10題, 每題十分.

1. 求 $\lim\limits_{x \to 0} \left(\dfrac{\sin x}{x}\right)^{1/x^2}$

2. 求 $\lim\limits_{x \to 1} [-x^2 + 2x + 2]$, 其中 $[a]$ 表不大於 a 之最大整數

3. 求函數 $g: [0,2] \to \mathbb{R}$: $g(x) = \sinh|x^2 + x - 2|$ 之最大值及最小值.

4. 若有半徑為 8 吋之一玉球, 欲容於一正圓錐体內, 試求此正圓錐体之高與底的尺寸, 可使正圓錐之体積為最小.

5. 求 $\displaystyle\int \dfrac{x \sin^{-1} x}{(1+x^2)^2}\, dx$

6. 求 $\displaystyle\int_{-1}^{1} \left([x] + [x + \tfrac{1}{2}]\right) dx$

7. 求由函數 $f(x) = x^2 e^{-x^2}$ 的圖形與其漸近線所圍區域之面積.

8. 求冪級數 $\displaystyle\sum_{n=0}^{\infty} \dfrac{(x+1)^n}{(n+1)2^n}$ 之收斂區間.

9. 求 $\sqrt{(3.01)^2 + (3.97)^2}$ 之近似值.

10. 求由三坐標面與平面 $20x + 12y + 15z = 60$ 所圍之四面体的体積.

(10 分) 1. Find all the solutions of the system of equations

$$
\begin{aligned}
2x_1 - x_2 + x_3 - x_4 &= 0 \\
x_1 - 3x_2 + 4x_3 - 3x_4 &= 5 \\
3x_1 - x_2 + 6x_3 - 4x_4 &= 1 \\
-x_1 + x_2 - 3x_3 + 2x_4 &= -5
\end{aligned}
$$

(10 分) 2. Find bases for (1) the column space and (2) the null space of the matrix

$$
\begin{pmatrix}
2 & 3 & 1 & -1 \\
5 & 2 & 1 & 3 \\
1 & 7 & 2 & -6 \\
6 & -2 & 0 & 8
\end{pmatrix}.
$$

(10 分) 3. Determine whether the 3×3 matrix

$$
\begin{pmatrix}
1 & 3 & -3 \\
2 & 5 & -3 \\
-2 & 2 & -4
\end{pmatrix}
$$

is invertible, and find its inverse if it is.

(15 分) 4. By the Gauss-Jordan reduction, one obtains the inverse of the 4×4 matrix A by a series of row operations: (1) interchanging the 1st row and the 3rd row (2) multiplying -3 to the 3rd row and add it to the 1th row (3) mutiplying the 2nd row by $\frac{1}{3}$ (4) add the 1st row to the 3th row and (5) interchanging the 1st row and the 4th row. Find A and A^{-1}.

(10 分) 5. Determine whether the 4×4 matrix A given by

$$
\begin{pmatrix}
-1 & 4 & 2 & -7 \\
0 & 4 & -3 & 6 \\
0 & 0 & -3 & -1 \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

is diagonalizable, and find a 4×4 matrix M such that $M^{-1}AM$ is diagonal if it is.

(15 分) 6. Find the rank of the 4×3 matrix A

$$
\begin{pmatrix}
1 & -1 & 0 \\
0 & 1 & 2 \\
2 & 1 & -3 \\
1 & -3 & 4
\end{pmatrix}.
$$

Can we find a 3×4 matrix B such that $BA = I_3$? Find such B if it can be done.

(30 分) 7. Let A be a matrix over $\mathbf{C}$.

　　a. What is the characteristic polynomial $P_A(x)$ of A?

　　b. What is the minimal polynomial $Q_A(x)$ of A?

　　c. Is $\deg P_A(x) = n$ if A is $n \times n$?

　　d. Is $\deg Q_A(x) = n$ if A is $n \times n$?

　　e. Let

$$
M = \left\{ A : P_A(x) = (x-2)^3 (x+1)^2 (x-1)^3 \right\}.
$$

Consider the relation "~" in M defined by

$A \sim B$ if $\exists$ invertible matrix $U \ni U^{-1}AU = B$.

Show that "~" is an equivalent relation on M and determine the number of equivalent classes in M.