應用數學系微積分(九十二學年轉學考)

Answer each of the following problems. Problems 1-4 carries 10 % each. Problems 5-8 carries 15 % each. The total is 100 %. Show details of your work.

(1) (10%) Evaluate the following

(a) $\frac{d}{dx}\tan^{-1}(5\tan x)$;

(b) $\lim_{x\to 0+} x^{\sin 2x}$.

(2) (10%) Evaluate the following integrals

(a) $\int x^5 e^{-x^3}\,dx$;

(b) $\int \frac{1}{\sqrt{-x^2+4x-3}}\,dx$.

(3) (10%) Let $f(x,y)=x-x^2-y^2$. Find a normal vector to the surface $\{(x,y,z): z=f(x,y)\}$ at $(1,1,-1)$.

(4) (10%) Find the interval of convergence for the series

$$\sum_2^\infty \frac{k^2x^k}{2^k}.$$

(5) (15%) Suppose a function f satisfies $|f(x)-f(y)|\le |x-y|^2$ for all $x,y\in\mathbf{R}$, show that f is a constant function.

(6) (15%) Prove that $\pi^e < e^\pi$. (Hint: Consider the function $\frac{\ln x}{x}$.)

(7) (15%) Find all values of $p,q\in\mathbf{R}$ such that the improper integral $\int_0^\infty \frac{x^p}{1+x^q}\,dx$ is convergent.

(8) (15%) Evaluate the triple integral

$$\iiint_S (1+x+y+z)^{-3}\,dV$$

where S is the region bounded by $x+y+z=1$, $x=0$, $y=0$ and $z=0$.

國立中山大學九十二學年度轉學生招生考試試題

科目：線性代數【應數系二年級】

1. (20分)Let $\alpha_1 = (-1,1,0,1,0)$, $\alpha_2 = (1,1,1,0,0)$, $\alpha_3 = (1,3,2,1,1)$, $\alpha_4 = (2,2,2,0,-1)$. $\beta_1 = (-1,3,0,2,1)$, $\beta_2 = (-1,1,-1,0,0)$, $\beta_3 = (-1,-1,1,2,2)$, $\beta_4 = (-1,0,0,1,1) \in \mathbb{R}^5$. Suppose that M_1 is (linear) spanned by $\alpha_1, \alpha_2, \alpha_3, \alpha_4$, and M_2 is spanned by $\beta_1, \beta_2, \beta_3, \beta_4$. Find the dimension and basis of the spaces M_1; M_2; $M_1 + M_2$ and $M_1 \cap M_2$.

2. (20分) $W = \{(x_1, x_2, x_3, x_4) \mid x_2 - x_4 = 0,\ 2x_1 - x_2 - 3x_3 = 0\}$

   (a) 試求出 W 的一組直交基底 (orthogonal basis).

   (b) 請求出 $\alpha = (1,0,1,1)$ 在 W 之投影向量.

3. (20分) Let V be a finite dimensional vector space and T is a linear transformation on V. Prove that T is one to one if and only if T is onto.

4. (20分) 空間中有兩平面 $(A): X + Y + Z = 0$, $(B): 2X - Y - Z = 0$
   請求出

   (a) 對 平面(A) 鏡射的方陣.

   (b) 對 平面(B) 鏡射的方陣.

   (c) 先對 平面(A) 鏡射, 再對 平面(B) 鏡射的方陣.

   (d) (c) 的結果會是繞某一軸之旋轉, 請問軸之方向與旋轉之角度.

5. (20分) 請畫出 $16X^2 - 24XY - 9Y^2 - 30X - 40Y = 0$ 之圖形.

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