(一) 填充題：每個空格4分，共計60分。此部份祇需將答案寫在答案卷上，並在答案前標明每個空格的英文字母代號；不需列出計算過程。

1. Evaluate the following limits :

   (i) $\lim_{x\to 0} \dfrac{2\cos x - 2 + x^2}{x^2 - \ln(1+x^2)} =$ __(A)__ (ii) $\lim_{n\to\infty} \sum_{k=1}^{n} \dfrac{1}{\sqrt{4n^2 - k^2}} =$ __(B)__

2. Evaluate the following integrals :

   (i) $\int_1^2 x\sqrt{2-x}\,dx =$ __(C)__ (ii) $\int_0^1 \{\int_{\sqrt{x}}^1 \dfrac{dy}{1+y^3}\}\,dx =$ __(D)__

3. Let p and q be two real numbers, and let $f(x) = e^{x^2-1} + px + q$ for all $x \in \mathbb{R}$. If $f(1) = 4$ is a local extremum of f, then $p =$ __(E)__ , $q =$ __(F)__
4. The length of the curve $y = \ln\cos x$ $(0 \le x \le \frac{\pi}{4})$, is __(G)__
5. Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a function. Assume that f is differentiable at the point $(1,2) \in \mathbb{R}^2$ with $f(1,2) = 0$, $f_x(1,2) = -1$ and $f_y(1,2) = 3$.
   (i) The equation of the tangent plane to the surface $z = f(x,y)$ at the point $(1,2,0)$ is __(H)__
   (ii) If $\vec{u} = (\frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}})$, then the directional derivative of f at $(1,2)$ in the direction $\vec{u}$ is __(I)__
   (iii) If $g(t) = f(1 - 4\tan^{-1} t, 3 - e^{2t})$ for $t \in \mathbb{R}$, then $g'(0) =$ __(J)__
6. If $\Omega = \{(x,y) \in \mathbb{R}^2 : 1 \le x \le 2, 0 \le y \le 1\}$, then

$$\iint_\Omega x e^{xy}\,dx\,dy = \underline{\quad(K)\quad}$$

7. If

$$f(x) = 3 + \int_2^{\sqrt{x}} \frac{dt}{1+3t^2+t^4} \quad \text{for } x > 0,$$

   and if g is the inverse of f, then $f'(x) =$ __(L)__ , and $g'(3) =$ __(M)__
8. The interval of convergence of the power series $\sum_{k=3}^{\infty} \frac{\ln k}{k}(x-2)^k$ is __(N)__
9. If $f(x) = \cos(x^2)$, then $f^{(12)}(0) =$ __(O)__

(二) 計算題：請詳列計算過程，否則不計分

I. Let

$$f(x) = \begin{cases} \frac{1-\cos x}{x} & \text{if } x \ne 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Prove that f is differentiable on $\mathbb{R}$, and find $f'(x)$. (10 %)

II. Evaluate the value of the integral :

$$\int_0^\infty \frac{dx}{(x^2+1)(x^2+4)} \quad (10\ \%)$$

III. Let $\Omega = \{(x,y) : x^2 + y^2 \le \frac{1}{4}$ and $x \ge 0\}$. Evaluate the double integral :

$$\iint_\Omega \sin^{-1}(x^2+y^2)\,dx\,dy \qquad (10\ \%)$$

IV. (i) If $g(t) = t^2 - \frac{t^4}{3} - \sin^2 t$, prove that $g(t) \le 0$ for all $t \in \mathbb{R}$. (5 %)

(ii) Use (i) to evaluate the limit : $\lim_{(x,y)\to(0,0)} \dfrac{\sin^2 x + \sin^2 y}{x^2+y^2}$ (5 %)

以下四題 每題 25分

(一) 求以下二個矩陣的反矩陣

(a) $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 2 & 1 & 5 \end{bmatrix}$　　(b) $\begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & 3 & -1 & 4 \\ 2 & 1 & 5 & -3 \\ 4 & 0 & 2 & 4 \end{bmatrix}$

(二) 設矩陣 A 為

$A = \begin{bmatrix} 0.8 & -0.2 & 0.4 \\ -0.2 & 0.6 & 0.5 \\ 0.4 & 0.5 & 0.3 \end{bmatrix}$　求 A 的 eigenvalue．說明為何它們均不大於 3．

(三) 請在 R^3 空間找一組 orthonormal basis (x_1, x_2, x_3), (y_1, y_2, y_3), (z_1, z_2, z_3) 使 x_i, y_i, z_i 均不為 0．並說明在 R^k 空間中如何做以上之問題？

(四) 設 $P=(1,0,-1)$, $Q=(1,1,1)$, $R=(2,2,1)$，回答以下問題

(a) 找一點 S 使 $PQRS$ 為平行四邊形，並求其面積

(b) 另外再找出 T, U, V 使 $OPQRSTUV$ 為一平行六邊形，並求其體積．$O=(0,0,0)$．