

1. Given a 8-bit binary number 10010011, answer each of the following problems. (20%)
 - (a) If it is an unsigned number, what is its value?
 - (b) If it is a signed number in ones complement, what is its value?
 - (c) If it is a signed number in twos complement, what is its value?
 - (d) If it is a BCD (binary coded decimal) number, what is its value?
2. If a computer will be connected to the Internet, some network settings have to be done, such as IP address, host name, domain name, gateway, subnet mask and etc.
 - (a) What is a gateway? What is its purpose? (5%)
 - (b) What is the purpose of the subnet mask? What is the relationship between the subnet mask and the IP address? Please explain with an example. (5%)
 - (c) What is a subnetwork of Class A in the Internet? What is a subnetwork of Class B in the Internet? What is the relationship between the IP address and Class A or B? It would be better if you can give some examples to explain. (5%)
 - (d) What is a proxy? What is its purpose? (5%)
3. What is Java? What is JavaScript? (8%)
4. What is the difference between a compiler and an interpreter? (12%)
5. What is the difference between a procedure call and a macro call? (12%)
6. The *quadratic selection sort* is described as follows: Divide the n input elements into k groups of k elements each, where $k = \sqrt{n}$. Find the largest element of each group and insert it into an auxiliary array. Find the largest of the elements in the auxiliary array. This is the largest element of all input elements. Then replace this element in the array by the next largest element of the group from which it came. Again find the largest element of the array. This is the second largest element of all input elements. Repeat the process until all elements have been sorted.
 - (a) Use the following 16 elements to explain how the algorithm works: 14, 5, 3, 8, 13, 15, 2, 9, 10, 4, 1, 6, 16, 7, 11, 12. (8%)
 - (b) How many comparisons are required for performing the quadratic selection sort if there are n input data elements, where $n = k^2$ for some integer k ? (7%)
7. Write a program, with FORTRAN, PASCAL or C, to multiply two matrices. (13%)

※ 請詳解下列每一個問題：

Problem 1. Evaluate the following limits :

$$(a) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cot x - \cos x}{(2x - \pi)^3} \quad (5\%) \quad (b) \lim_{x \rightarrow \infty} x \left(\frac{\pi}{2} - \tan^{-1} x \right) \quad (5\%)$$

Problem 2. Let C be the plane curve of the equation :

$$x^2 + xy - 2y^3 = 0.$$

(a) Find the equation of the tangent line to C at the point $(1, 1)$. (6%)

(b) Find $\frac{d^2y}{dx^2}$ at the point $(1, 1)$. (7%)

Problem 3. Let

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x < 0, \\ 2x & \text{if } x \geq 0. \end{cases}$$

(a) Is f continuous at $x = 0$? Prove your answer. (7%)

(b) Is f differentiable at $x = 0$? Prove your answer. (8%)

Problem 4. Let a and b be real numbers with $a < b$. Let f be a function continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) . If $f'(x) = 0$ for all $a < x < b$, prove that f is a constant function on $[a, b]$. (10%)

Problem 5. Evaluate the following integral :

$$\int_0^1 x^2 \ln x \, dx \quad (10\%)$$

Problem 6. Find the length of the curve $y = \frac{1}{2}x^2$ for $0 \leq x \leq 1$. (10%)

Problem 7. Let $f(x) = e^x \sin x$, $0 \leq x \leq \frac{\pi}{2}$, and let Ω be the region bounded by the graph of f , the x -axis and the vertical line $x = \frac{\pi}{2}$. Find the volume of the solid generated by revolving Ω about the x -axis. (10%)

Problem 8. Use the definition to prove :

$$\lim_{x \rightarrow 1} \frac{2x + 1}{x^2 + 1} = \frac{3}{2} \quad (7\%)$$

Problem 9. Let

$$f(x) = -3 + \int_0^{x+x^3} \frac{dt}{\sqrt{1+t^4}} \quad \text{for all real numbers } x.$$

(a) Prove that f is one-to-one. (5%)

(b) Let g be the inverse of f . Find $g'(-3)$. (5%)

(c) Does the limit of f at ∞ exist? Prove your answer. (5%)

* 請詳解下列每一個問題：

Problem 1. Let $f(x, y) = \sin(xy)$.

- (a) Find the gradient $\nabla f(1, \frac{\pi}{3})$ of f at the point $(1, \frac{\pi}{3})$. (5 %)
- (b) Find the equation of the tangent plane to the graph of f at the point $(1, \frac{\pi}{3}, f(1, \frac{\pi}{3}))$. (5 %)
- (c) Let $\vec{u} = (-3, 4)$ be a vector. Find the directional derivative of f at the point $(1, \frac{\pi}{3})$ in the direction of \vec{u} . (5 %)

Problem 2. Let $f(x, y) = -x^3 + 4xy - 2y^2 + 1$. Find the relative extrema of f , and find saddle points of the graph of f . (10 %)

Problem 3. Let $\Omega = \{(x, y) : x^2 + y^2 \leq 1 \text{ and } y \geq 0\}$. Evaluate the double integral :

$$\iint_{\Omega} \tan^{-1}(x^2 + y^2) dx dy \quad (10 \%)$$

Problem 4. Evaluate the iterated integral :

$$\int_0^1 \int_x^1 \frac{1}{1+y^4} dy dx \quad (10 \%)$$

Problem 5. Let C be the plane curve of the equation $x^4 + 4y^4 = 1$. Find the points on C which are closest to the origin. (10 %)

Problem 6. Let $f(x, y) = \begin{cases} \frac{\sin(x^2) + \sin(y^2)}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0), \\ 1 & \text{if } (x, y) = (0, 0). \end{cases}$

- (a) Is f continuous at $(0, 0)$? Prove your answer. (6 %)
- (b) Evaluate $f_x(x, y)$ for all (x, y) . (8 %)
- (b) Evaluate $f_{xy}(0, 0)$. (5 %)

Problem 7. Determine the convergence or divergence of the series

(a) $\sum_{n=1}^{\infty} n^2(1 - \cos \frac{1}{n})$ (8 %) (b) $\sum_{n=1}^{\infty} \left\{ \frac{2n}{2n+1} \right\}^{n^2}$ (8 %)

Problem 8. Find the interval of convergence of the power series :

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{2^n \cdot n} \quad (10 \%)$$