中山應數系 碩士班甄試
丙組第一梯次

Please prepare and solve ~~as many problems as you can~~ ONE OR TWO PROBLEMS during your interview.

1. Let $A \in M_{3\times 4}(\mathbf{R})$. Find the necessary and sufficient conditions for the existence of some $B \in M_{4\times 3}(\mathbf{R})$ such that $AB = I_3$.

2. (a) Define limit supremum and limit infimum of a sequence in $\mathbb{R}$. Give some of their equivalent statements.

   (b) Evaluate $\limsup\limits_{n\to\infty} \dfrac{1}{n}\{1-2+3-4+\cdots+(-1)^{n-1}n\}$.

3. Let Ω be an open and bounded subset of $\mathbb{R}^2$.

   (a) Fix $\mathbf{x} \in \mathbb{R}^2$, is the function $f(\mathbf{y}) = \|\mathbf{x}-\mathbf{y}\|$ continuous on $\mathbb{R}^2$?

   (b) For any $\mathbf{x} \in \Omega$, define $d(\mathbf{x}, \partial\Omega) \triangleq \inf\{\|\mathbf{x}-\mathbf{y}\| : \mathbf{y} \in \partial\Omega\}$. Show that there exists $\mathbf{y} \in \partial\Omega$ such that $d(\mathbf{x}, \partial\Omega) = \|\mathbf{x}-\mathbf{y}\| > 0$.

   (b) Would part (b) be true when Ω is unbounded?

4. Please find (and explain the reasons):

   (a) all elements of $\mathbf{Z}_{2012}$ with order 3.

   (b) all elements of $\mathbf{Z}_{2012}$ with order 4.

   (c) all subgroups of $\mathbf{Z}_{2012}$ with order 3.

   (d) all subgroups of $\mathbf{Z}_{2012}$ with order 4.

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Please prepare and solve ~~as many problems as you can~~ ONE OR TWO PROBLEMS during your interview.

1. (a) State the Taylor expansion theorem.

   (b) Apply Taylor expansion theorem to show that the Maclaurin series of the function $f(x) = e^x$ is convergent.

2. Let $f : \mathbb{R}^n \to \mathbb{R}$.

   (a) What is the definition of f being differentiable at a point $\mathbf{x}_0 \in \mathbb{R}^n$?

   (b) Let $f(\mathbf{x}) = \|\mathbf{x}\|^{-1}$. Give the direction for steepest descent of f at a point $\mathbf{x}_0 \neq \mathbf{0}$.

3. Prove or disprove: $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ are similar.

4. Suppose that $P, Q \in M_n(\mathbf{R})$ are similar. Prove or disprove:

   (a) characteristic polynomial of P = characteristic polynomial of Q.

   (b) set of eigenvectors of P = set of eigenvectors of Q.

   (c) trace of P = trace of Q.